The role of non-ideal magnetohydrodynamics in the formation of stars and their discs

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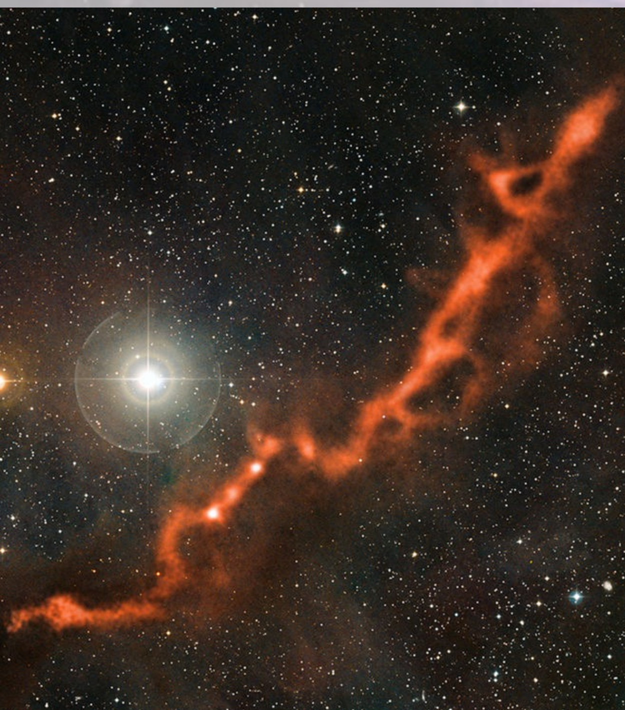
Star formation: From the beginning

Orion Molecular Cloud. (image credit: Drudis & Goldman via APOD)

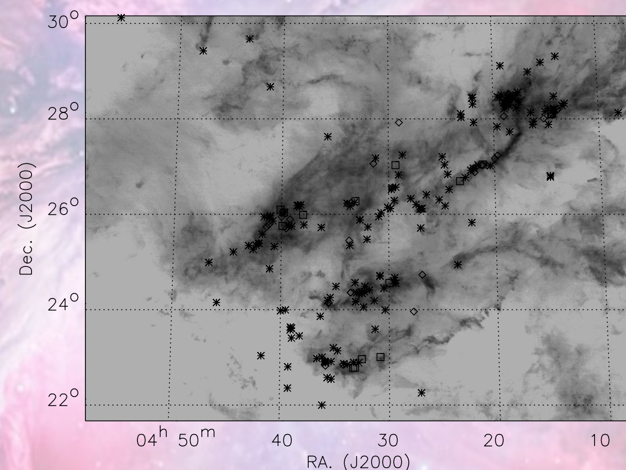
Star formation: From the beginning

Pillars of Creation: Hubble Space Telescope [visible] vs JWST [near IR] vs JWST [mid-IR].
(image credit: webbtelescope.org)

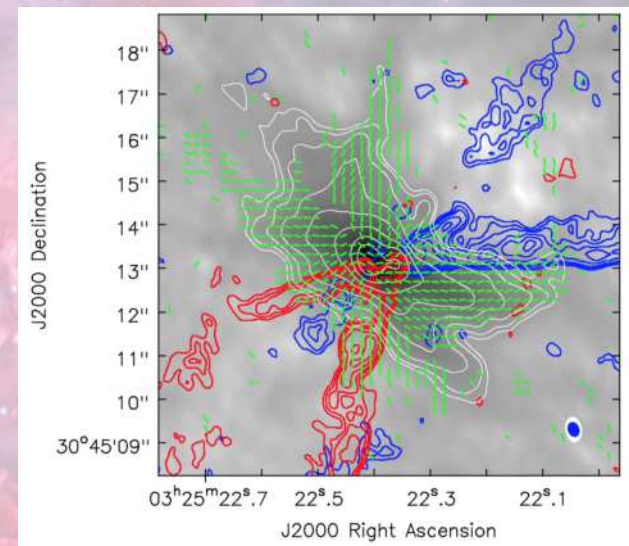
Star formation: Stellar nurseries



Taurus Molecular Cloud (Credit: ESO/APEX (MPIfR/ESO/OSO)/A. Hacar et al./Digitized Sky Survey 2. Acknowledgment: Davide De Martin)

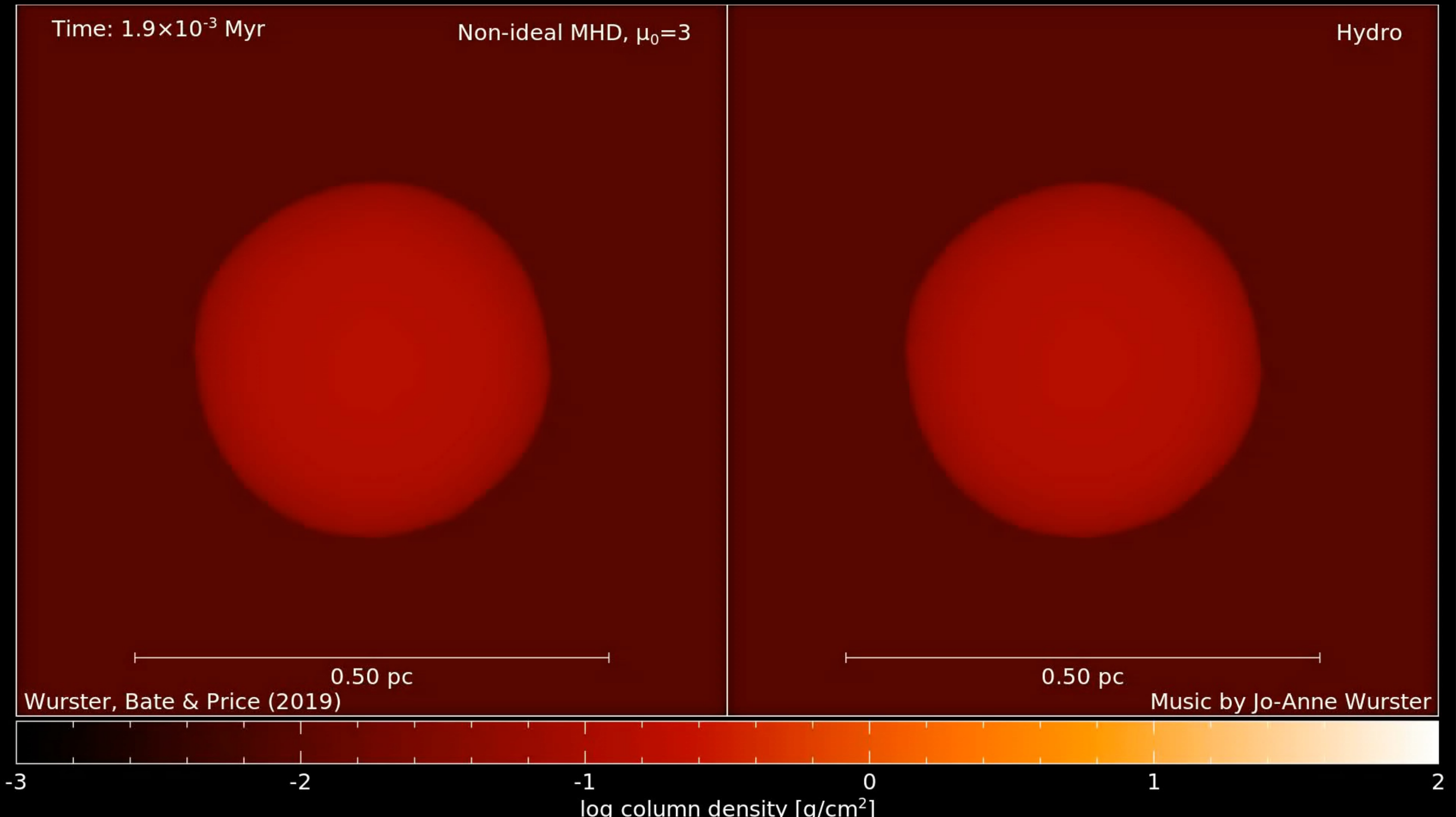


Taurus Molecular Cloud: H_2 column density map with positions of young stars (Goldsmith et. al., 2008)



Magnetic field morphology around L1448 IRS 2 (Kwon+ 2019)

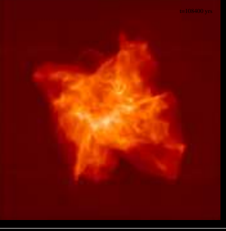
Cluster Formation: Effect of MHD



Available at https://youtu.be/VZixbkDMZO8

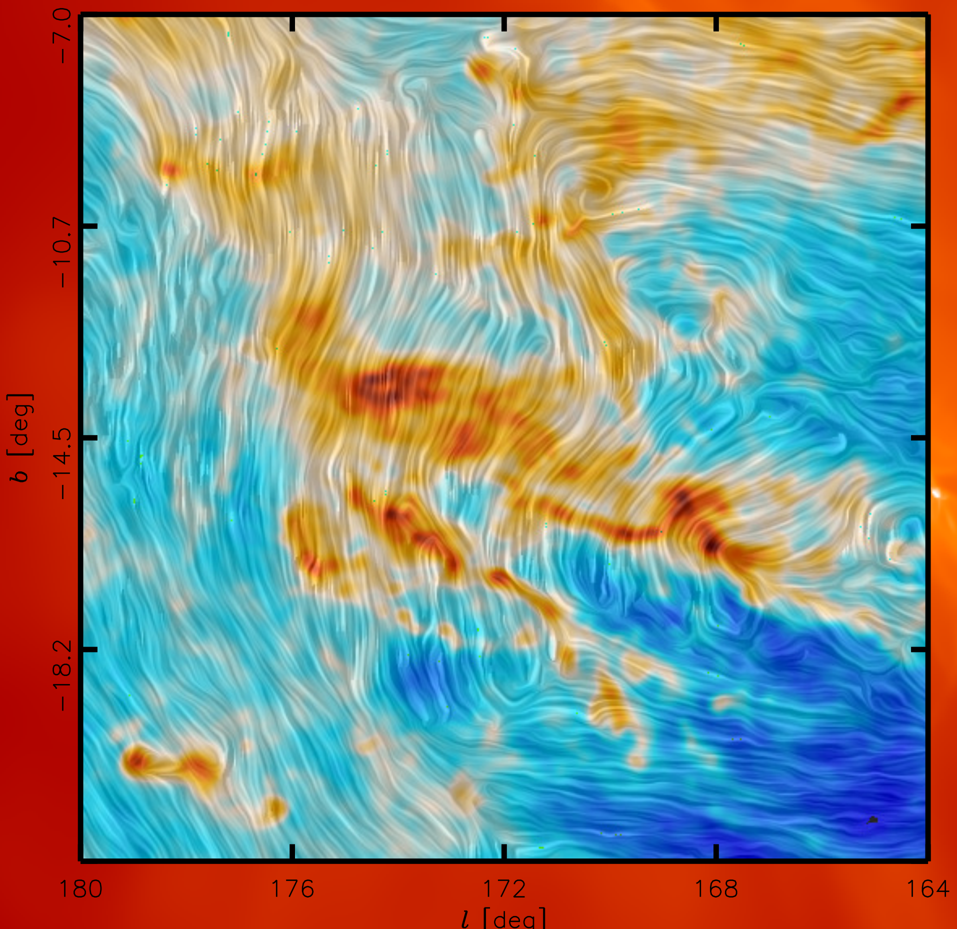
Wurster, Bate & Price (2019)

Music by Jo-Anne Wurster

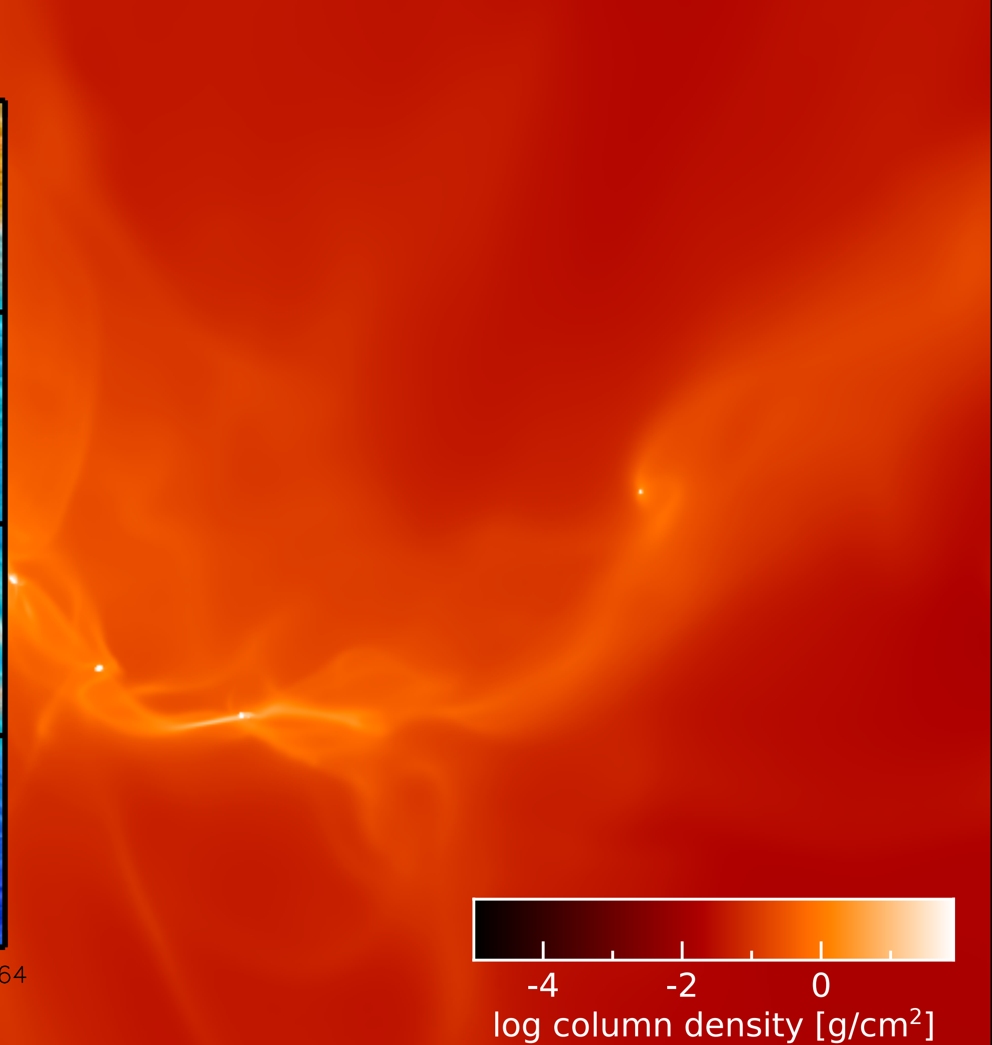


Magnetic fields in star forming regions

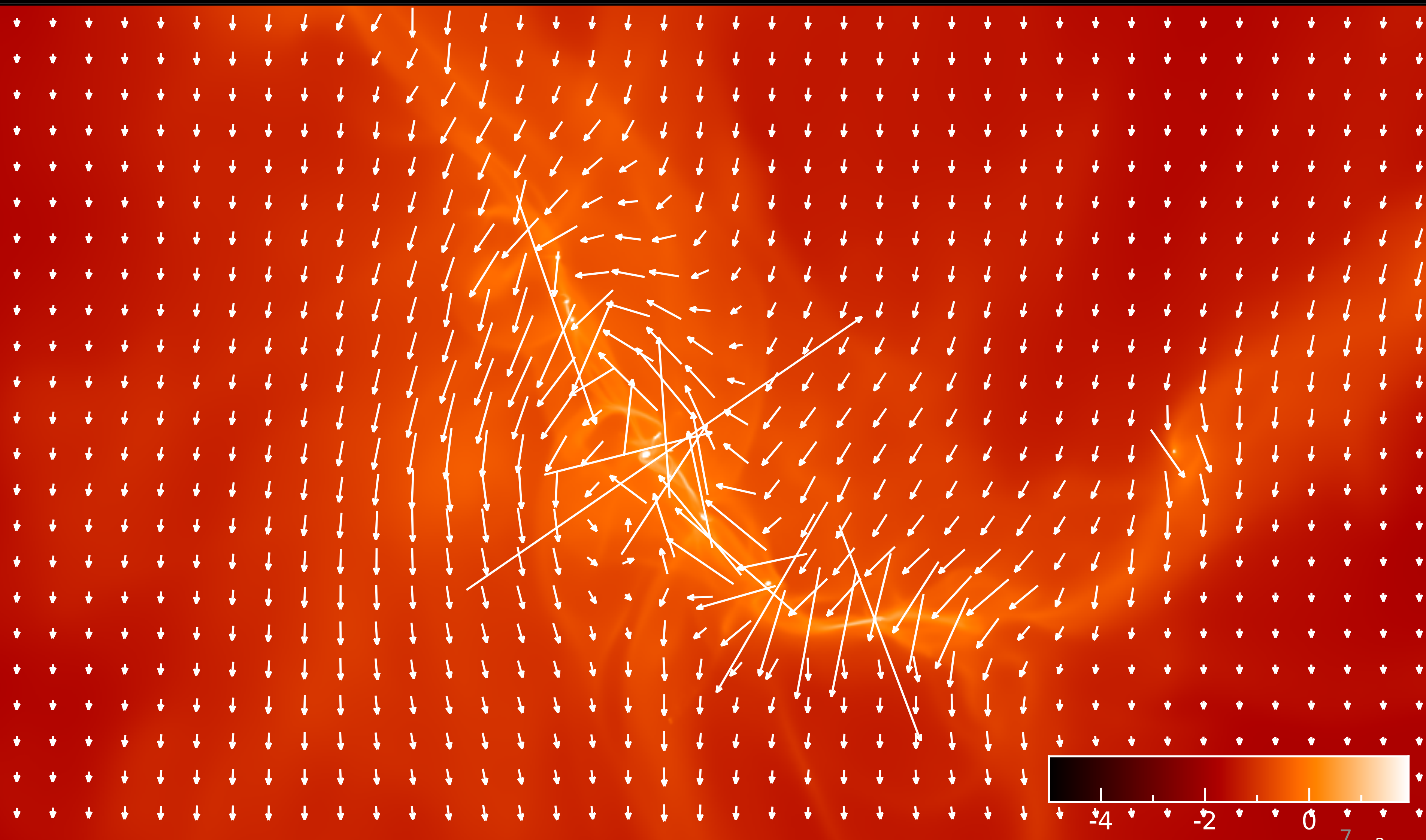
- Large-scale magnetic fields are perpendicular to dense structures
- Large-scale magnetic fields are parallel to low-density structures



Planck Collaboration (2016)

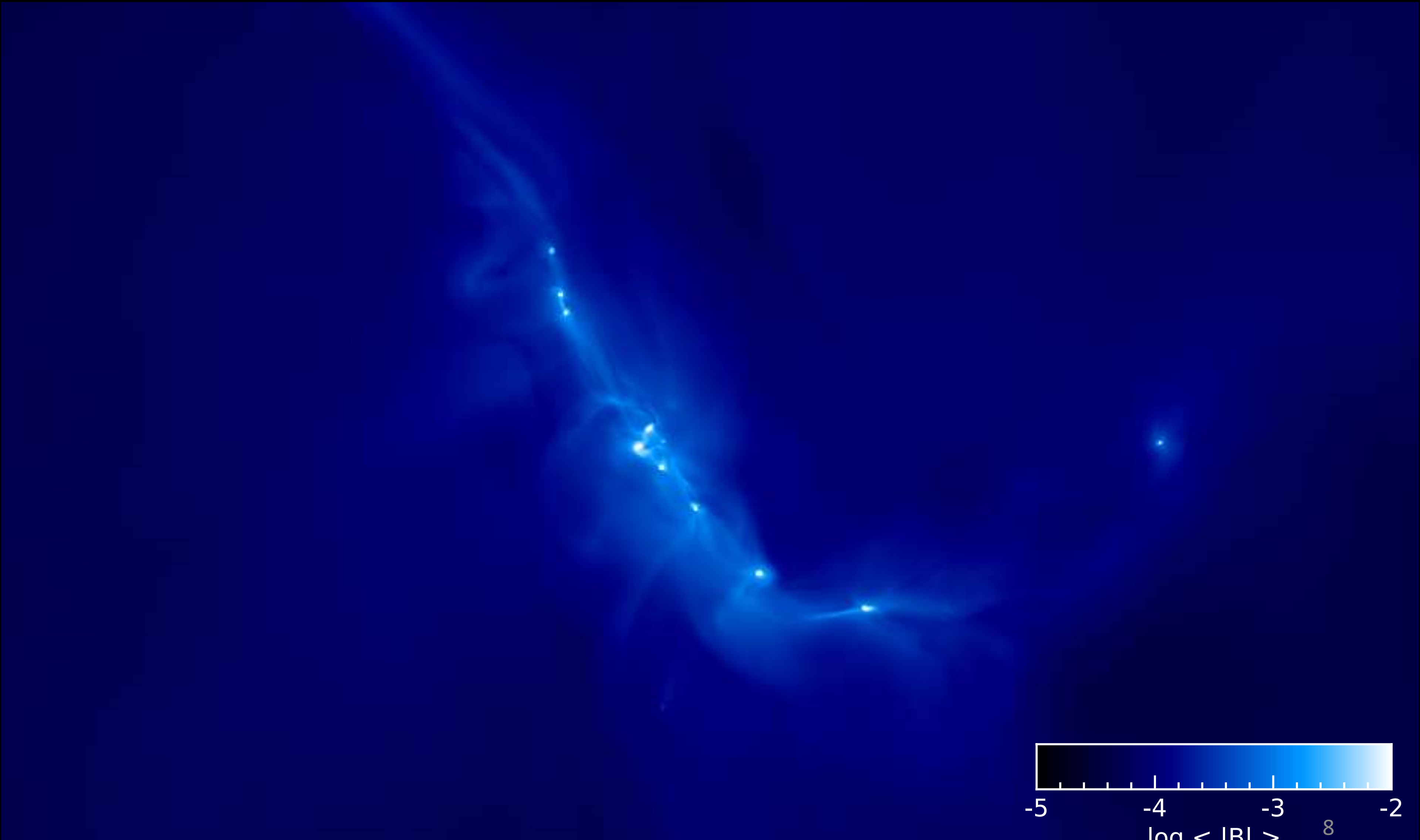


Magnetic fields in star forming regions



Data from Wurster, Bate & Price (2019)

Magnetic fields in star forming regions



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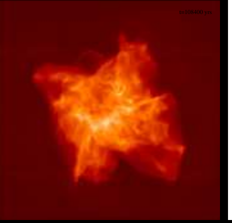
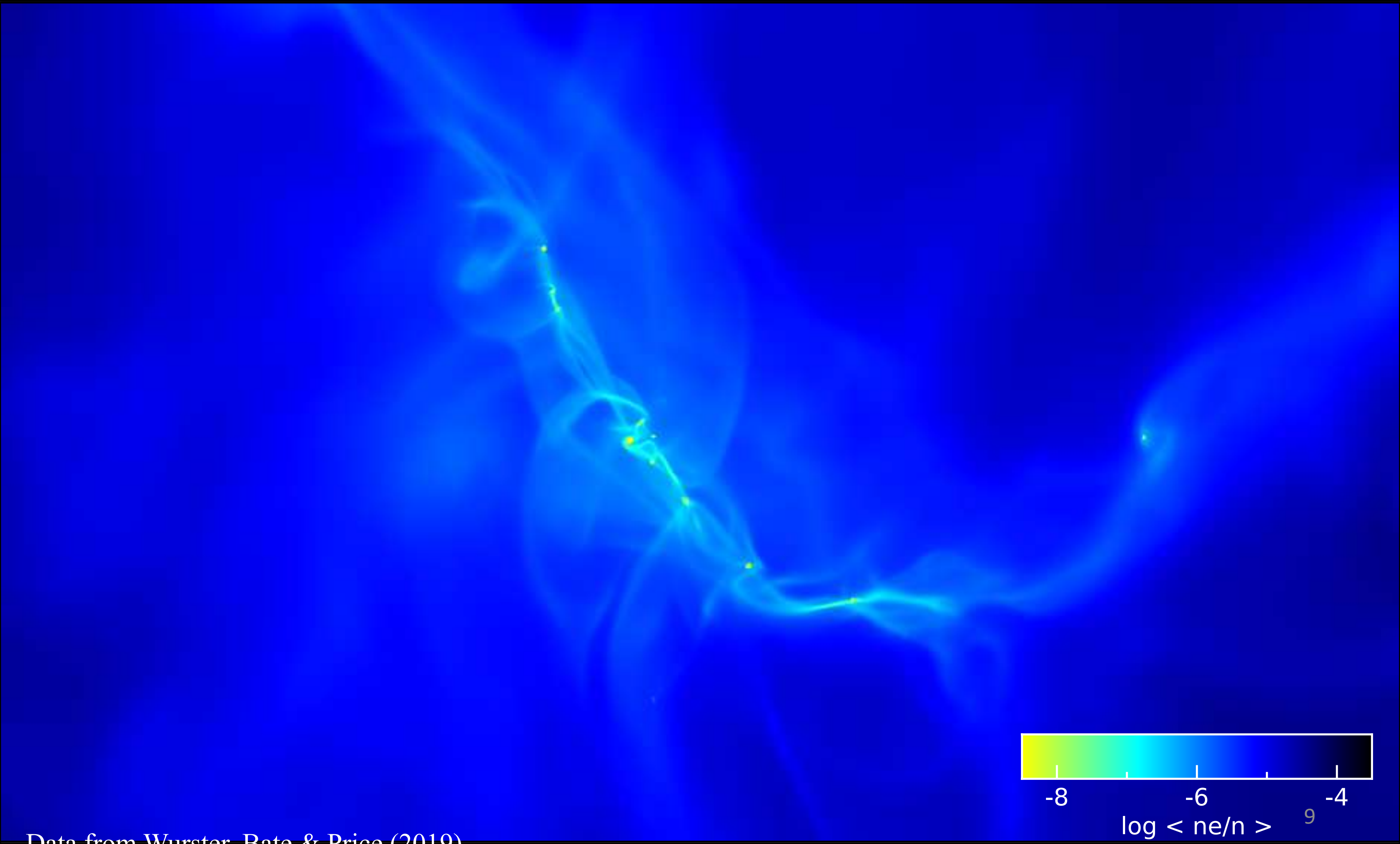


Magnetic fields in star forming regions: Ionisation fraction



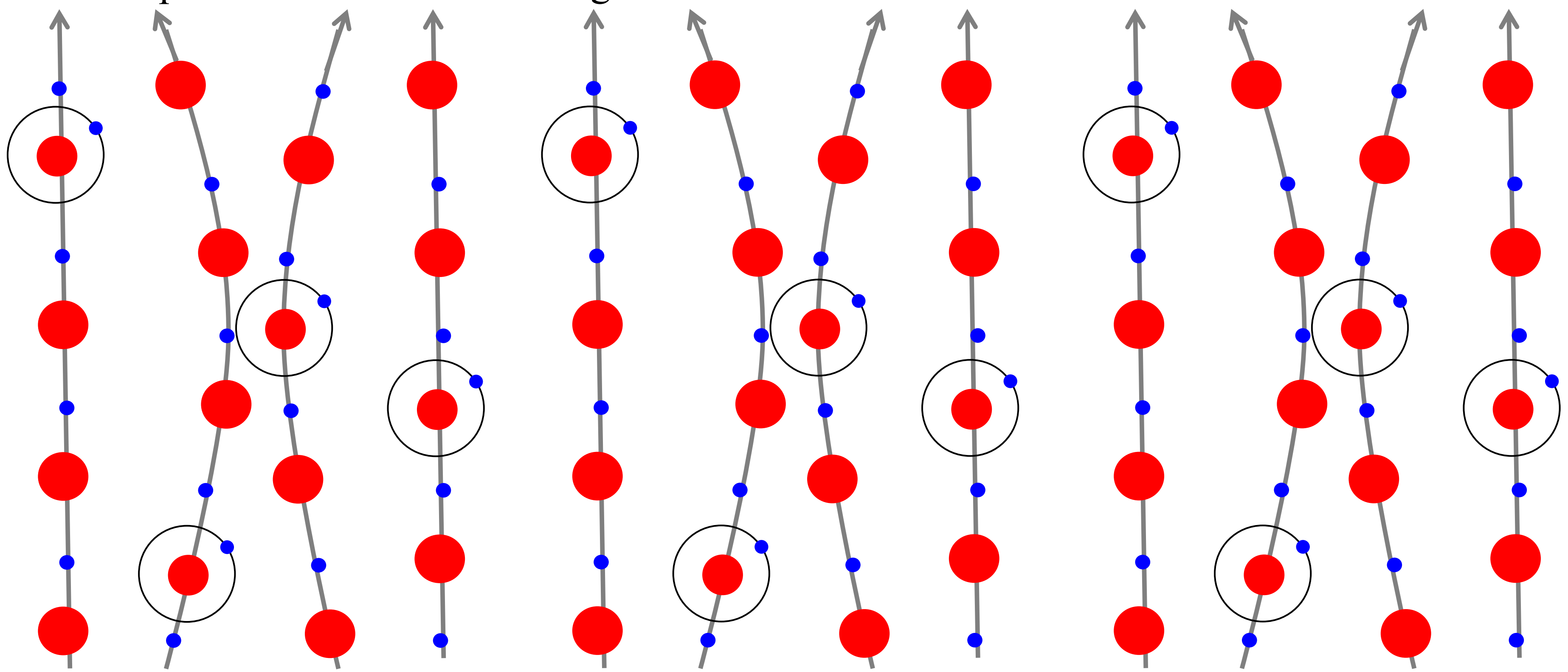
Data from Wurster, Bate & Price (2019)

Ideal magnetohydrodynamics

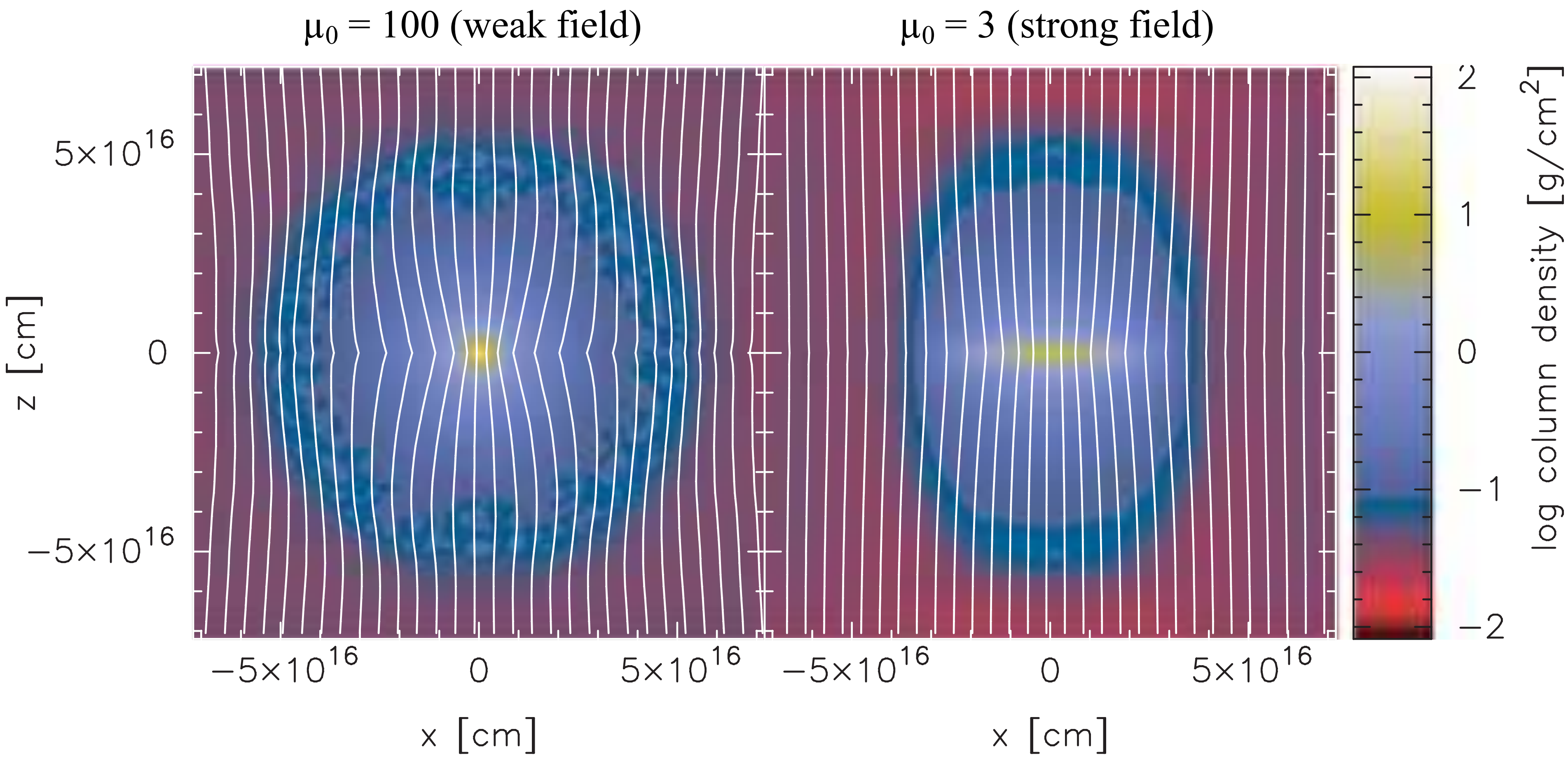
- Highly ionised plasma:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$$

- Zero resistivity & infinite conductivity
- Ions & electrons are tied to the magnetic field
- Neutral particles are tied to the magnetic field due to interactions with the ions & electrons



Ideal magnetohydrodynamics



Price & Bate (2007)

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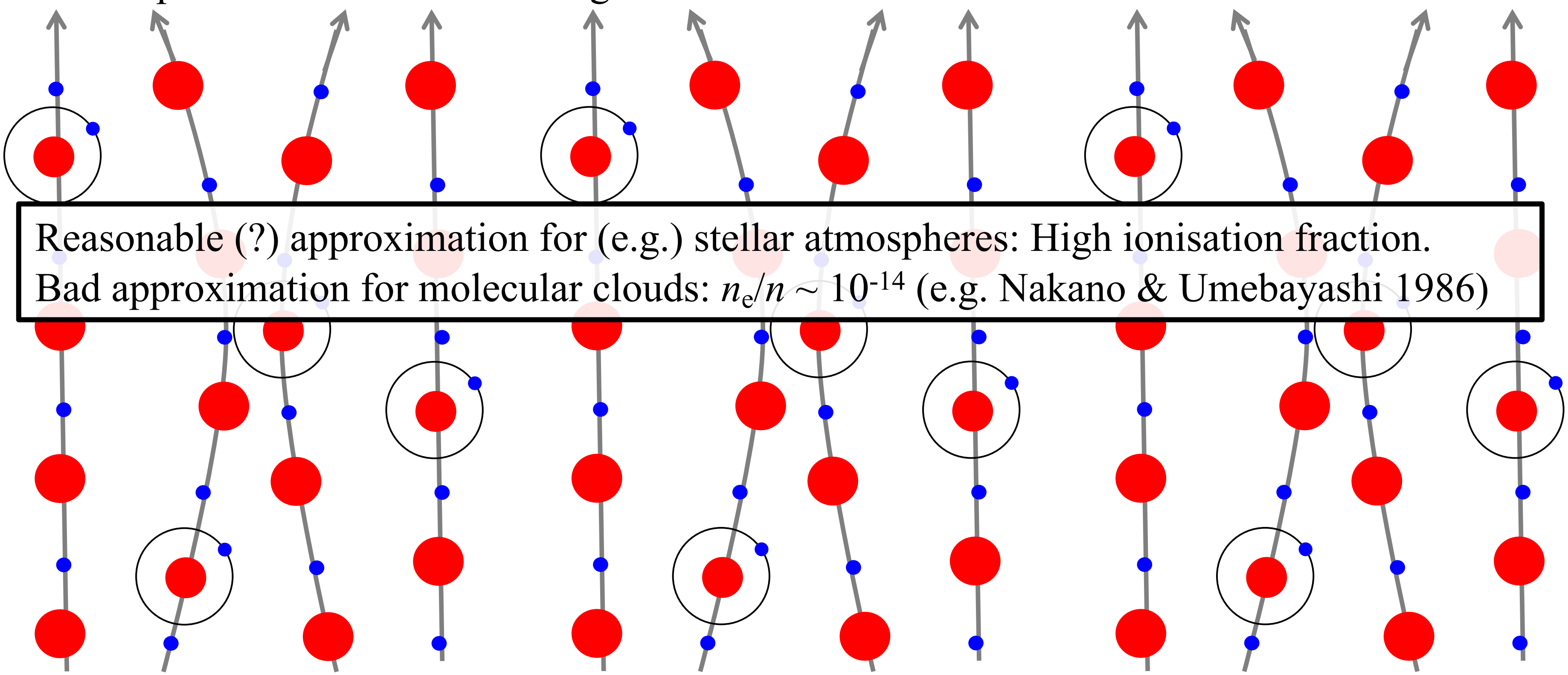
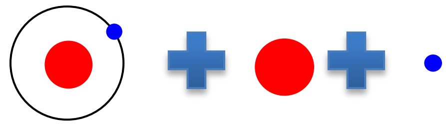


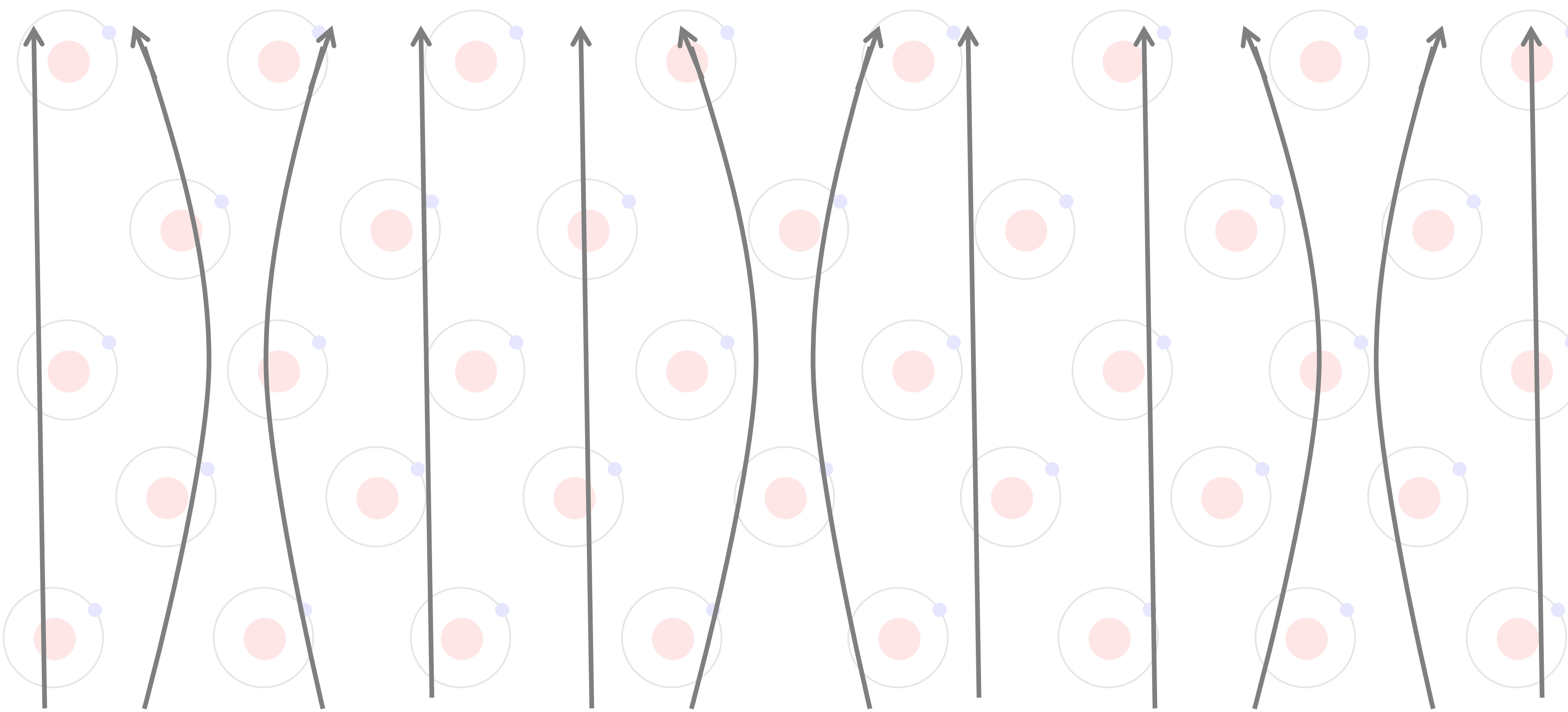
Reasonable (?) approximation for (e.g.) stellar atmospheres: High ionisation fraction.
Bad approximation for molecular clouds: $n_e/n \sim 10^{-14}$ (e.g. Nakano & Umebayashi 1986)

Non-ideal magnetohydrodynamics

- Partially ionised plasma:



- Non-zero resistivity & conductivity
- Ions, electrons & neutrals behaviour is environment-dependent



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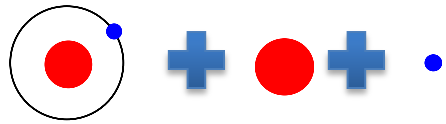
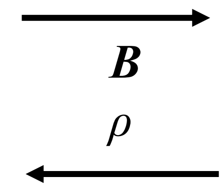
- Partially ionised plasma: 



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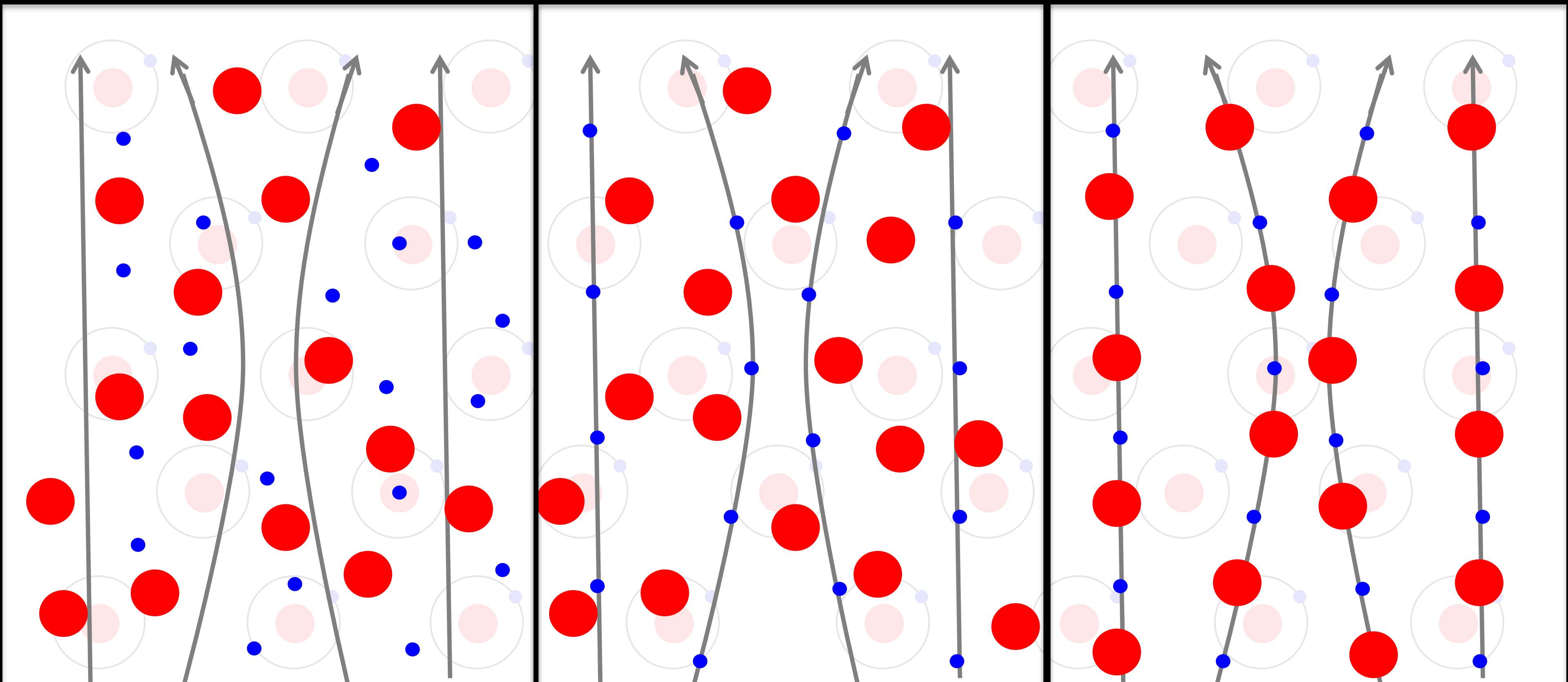
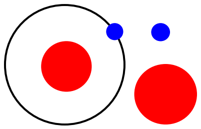
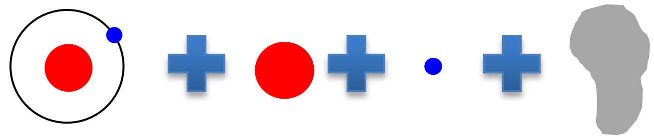
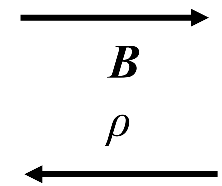


Ohmic Resistivity (diffusive) | Hall Effect (dispersive) | Ambipolar Diffusion (diffusive)



Non-ideal magnetohydrodynamics

- Partially ionised plasma and dust: 



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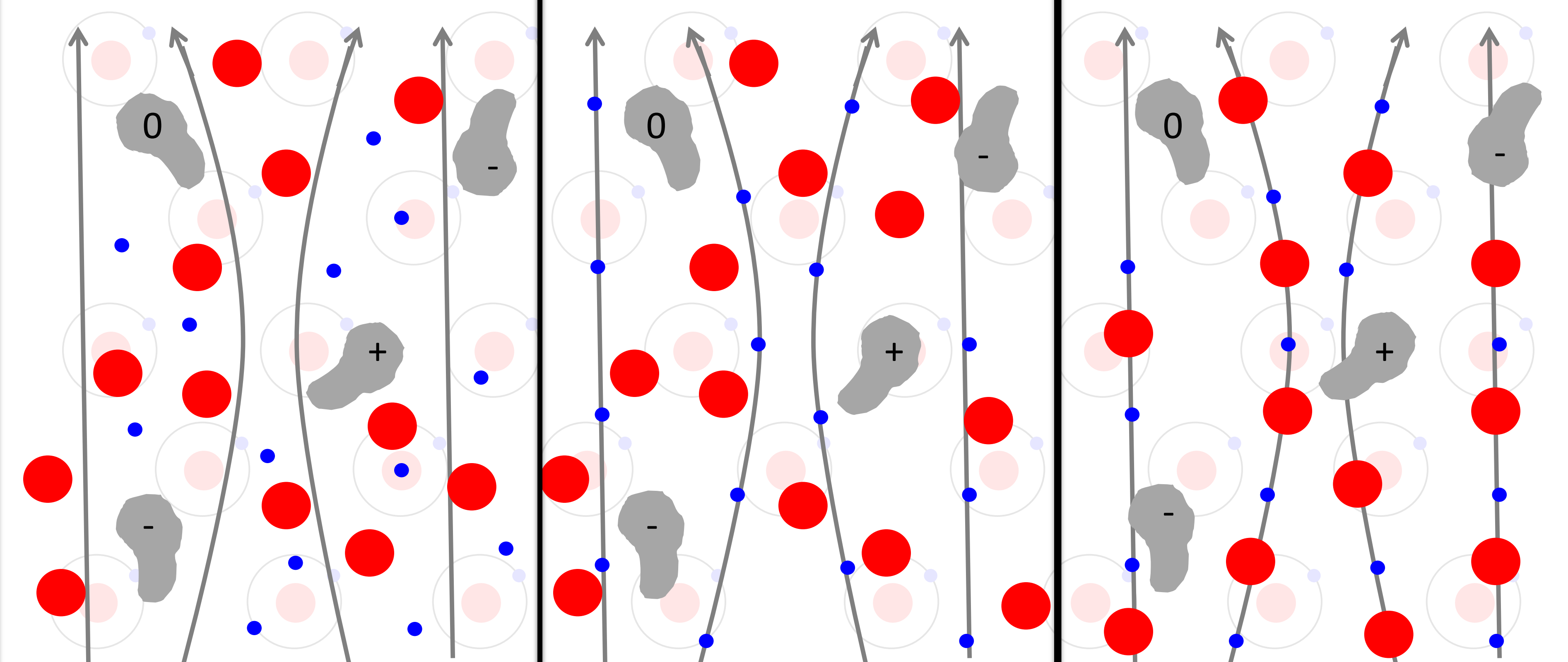
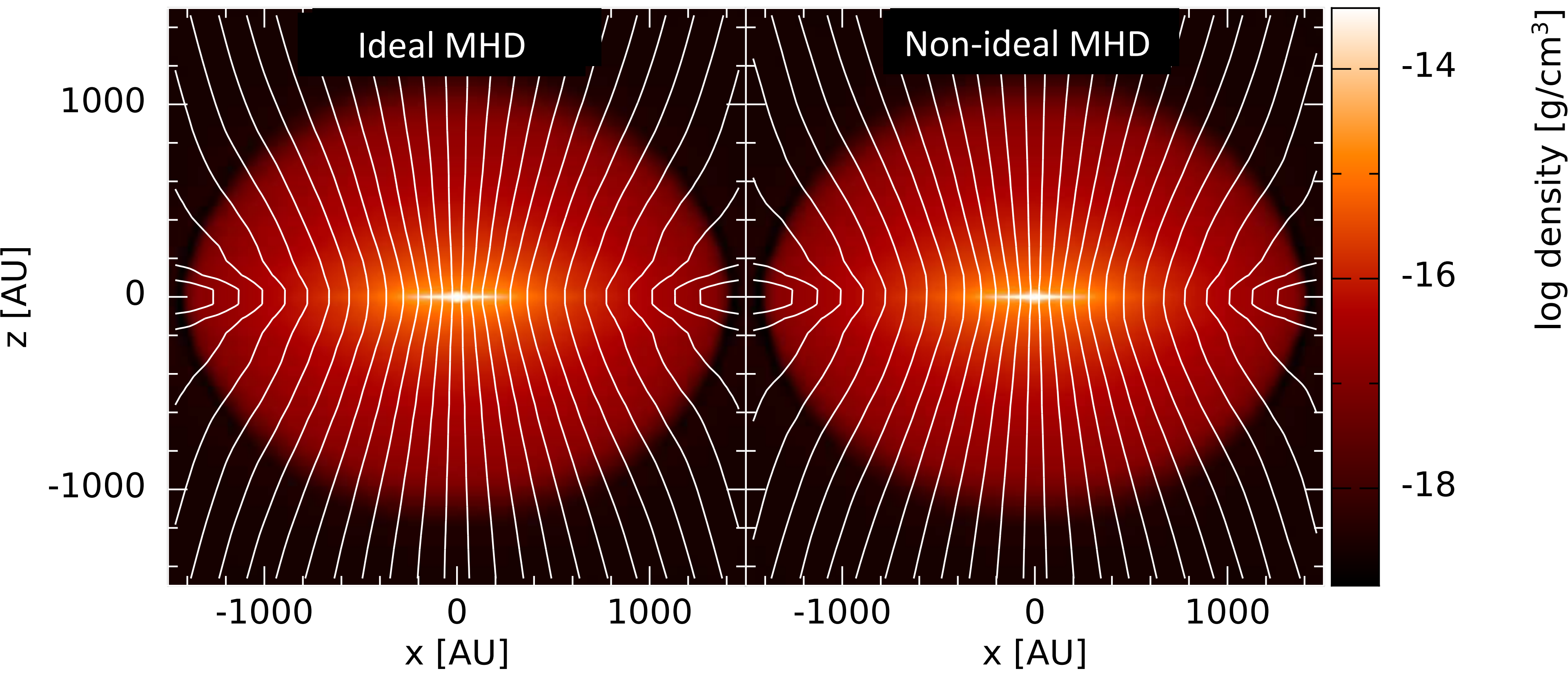


Ohmic Resistivity (diffusive) | Hall Effect (dispersive) | Ambipolar Diffusion (diffusive)

Non-ideal magnetohydrodynamics

- Strong field, initially vertical magnetic field
- Large scale structure



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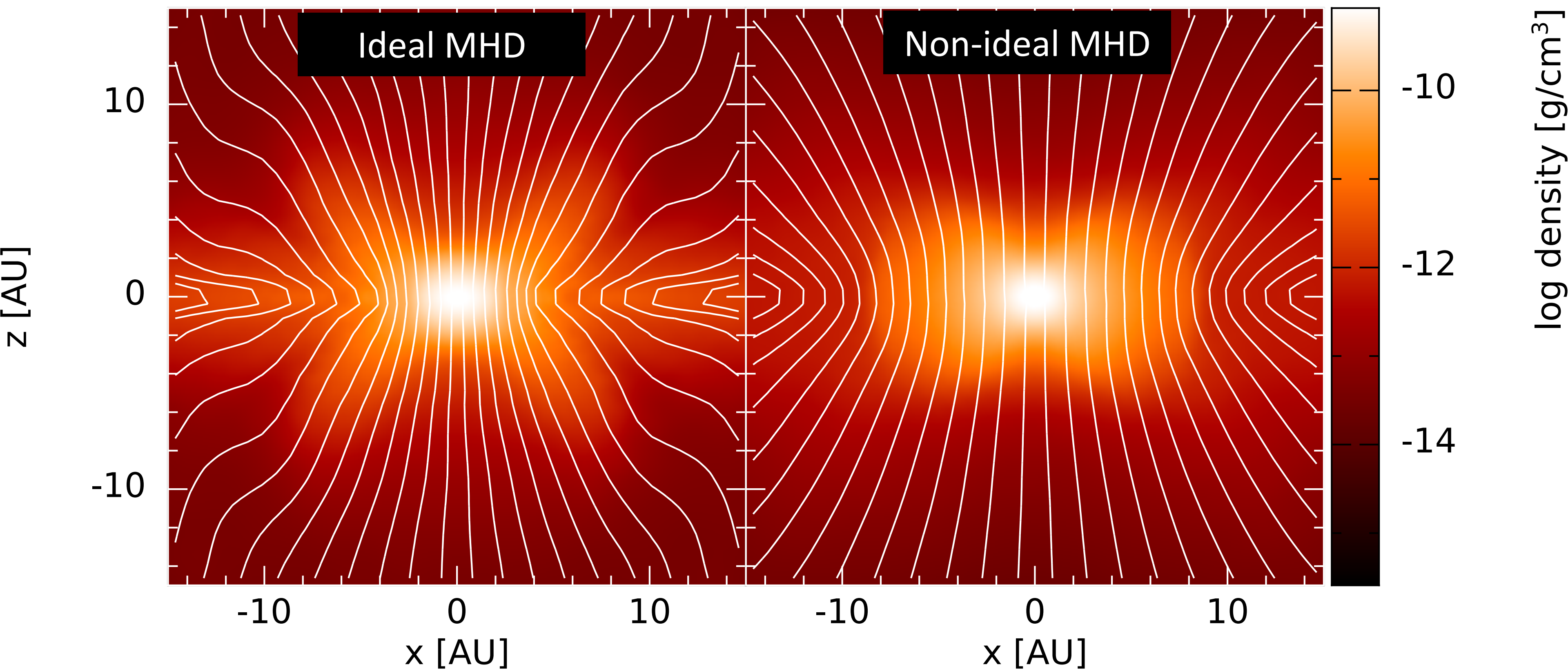

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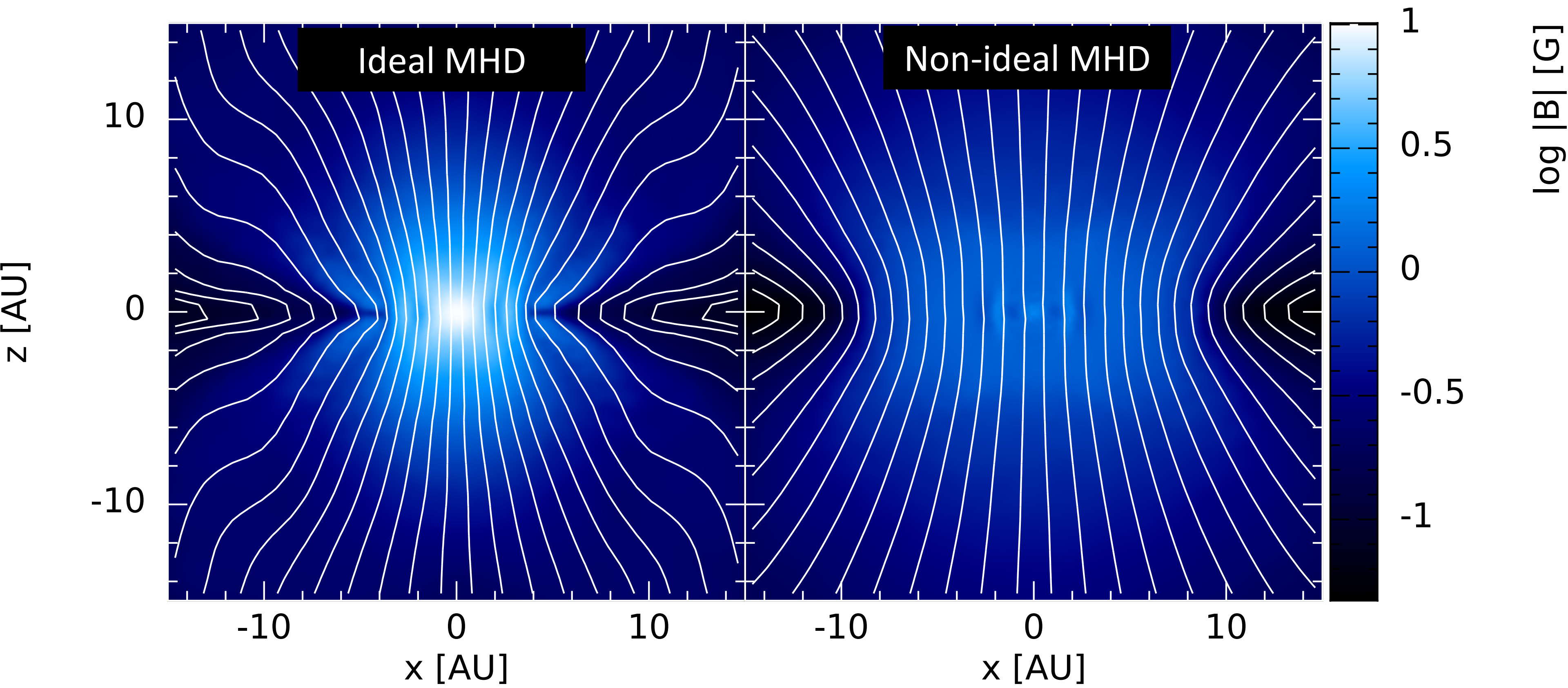
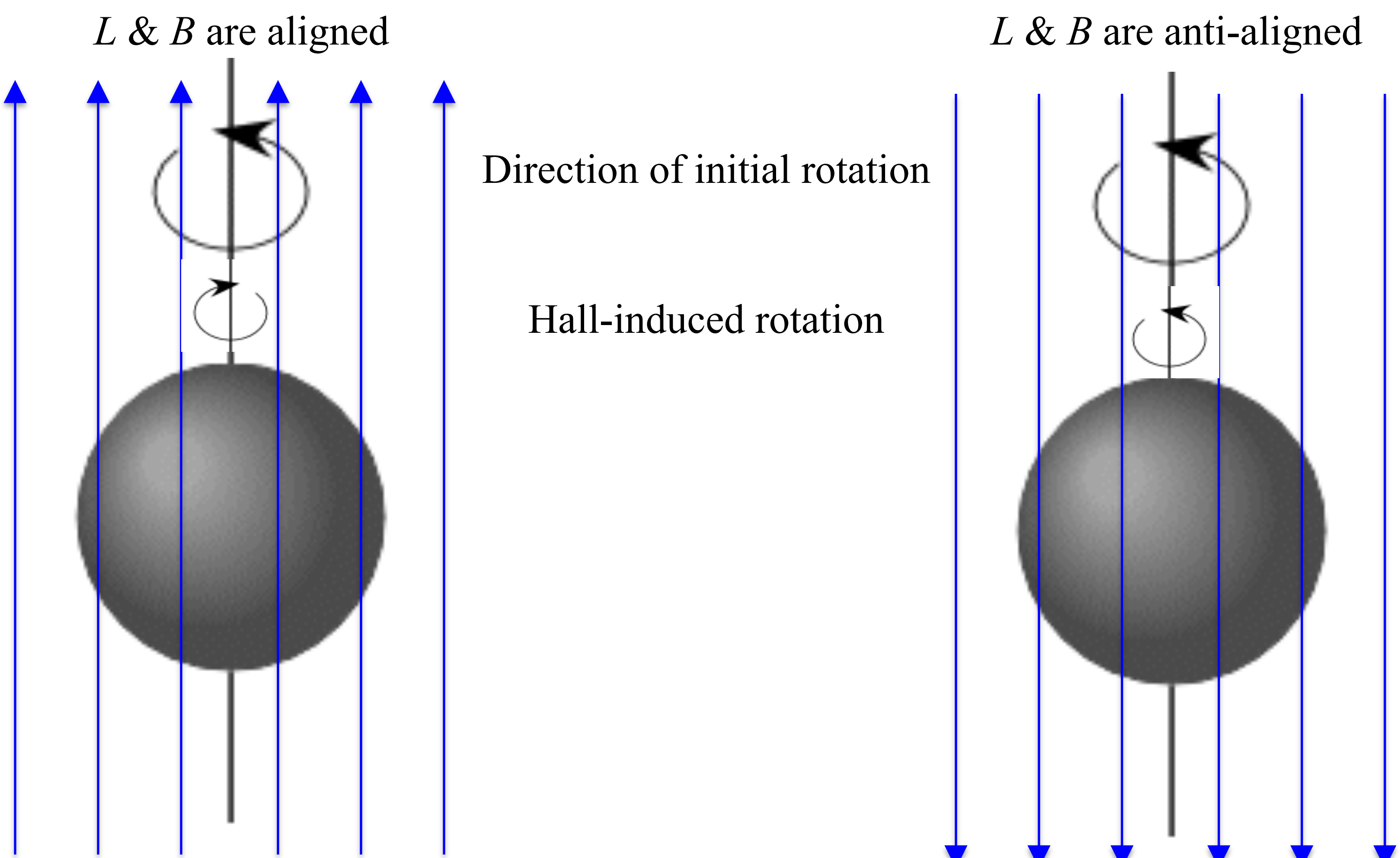


Non-ideal magnetohydrodynamics: Hall effect

➢Depending on the relative orientation of *L* & *B*, the Hall-induced rotation will contribute to or detract from the initial rotation



see also: Braiding & Wardle (2012a,b)

Continuum Magnetohydrodynamic Equations

- Continuum equations:

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \boldsymbol{v}$$

$$\frac{d\boldsymbol{v}}{dt} = -\frac{1}{\rho} \nabla \cdot \left[\left(p + \frac{B^2}{2} \right) I - \boldsymbol{B}\boldsymbol{B} \right] - \nabla \Phi + \frac{\kappa F}{c}$$

$$\rho \frac{d}{dt} \left(\frac{\boldsymbol{B}}{\rho} \right) = (\boldsymbol{B} \cdot \nabla)\, \boldsymbol{v} + \left. \frac{d\boldsymbol{B}}{dt} \right|_{\text{non-ideal}}$$

$$\rho \frac{d}{dt} \left(\frac{E}{\rho} \right) = -\nabla \cdot \boldsymbol{F} - \nabla \boldsymbol{v} : \boldsymbol{P} + 4\pi \kappa \rho B_P - c \kappa \rho E$$

$$\rho \frac{du}{dt} = -p \nabla \cdot \boldsymbol{v} - 4\pi \kappa \rho B_P + c \kappa \rho E + \rho \left. \frac{du}{dt} \right|_{\text{non-ideal}}$$

$$\nabla^2 \Phi = 4\pi G \rho$$

- Relevant processes:
  - Gas
  - ~~Dust~~
  - Radiation
  - Magnetic fields
  - Kinematics

Continuum Magnetohydrodynamic Equations

- Non-ideal MHD terms hide considerable micro-physics:

$$\left.\frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}t}\right|_{\text{non-ideal}} = -\nabla \times [\eta_{\text{OR}} (\nabla \times \boldsymbol{B})]$$

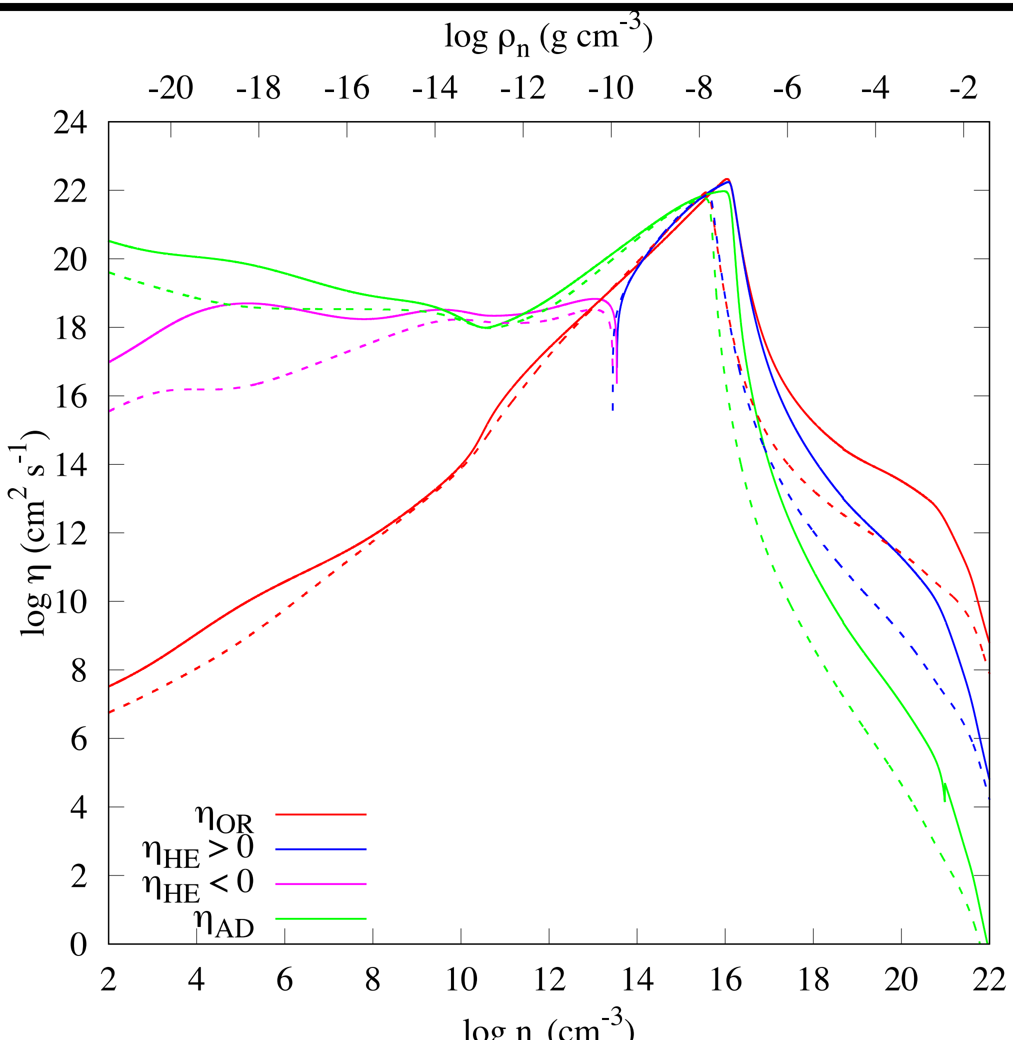
$$- \nabla \times \left[\eta_{\text{HE}} (\nabla \times \boldsymbol{B}) \times \hat{\boldsymbol{B}}\right]$$

$$+ \nabla \times \left\{\eta_{\text{AD}} \left[(\nabla \times \boldsymbol{B}) \times \hat{\boldsymbol{B}}\right] \times \hat{\boldsymbol{B}}\right\}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t}_{\text{non-ideal}} = \frac{\eta_{\text{OR}}}{\rho} |\nabla \times \boldsymbol{B}|^2$$

$$+ \frac{\eta_{\text{AD}}}{\rho} \left\{|\nabla \times \boldsymbol{B}|^2 - \left[(\nabla \times \boldsymbol{B}) \cdot \hat{\boldsymbol{B}}\right]^2\right\}$$

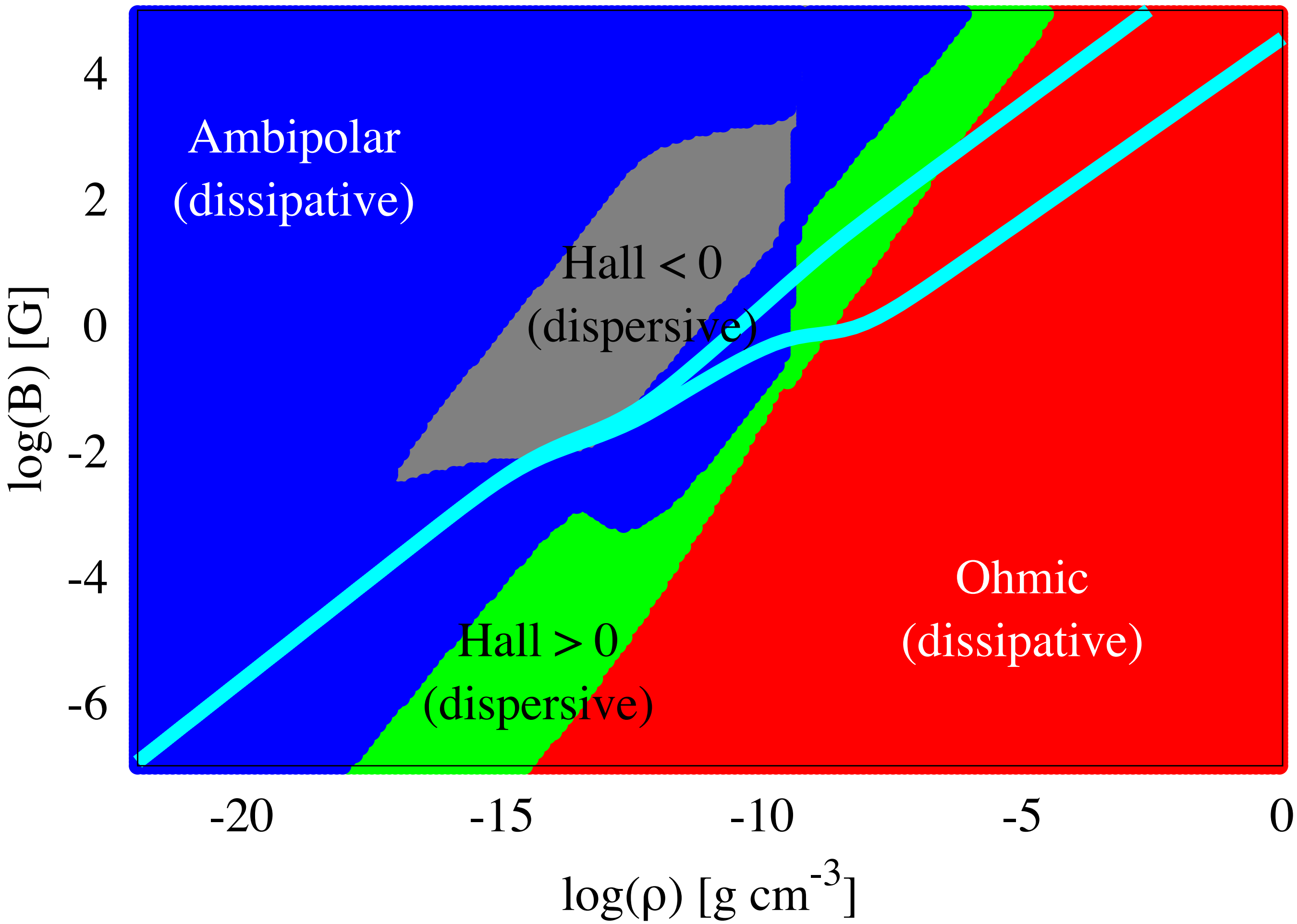
Magnetic fields in star forming regions: Non-ideal Effects



- Values dependent on microphysics: Grain size, ionised species, cosmic ray ionisation rate
- Solid: NICIL v2.1; dotted: NICIL v1.2.6.

Wurster (2021)

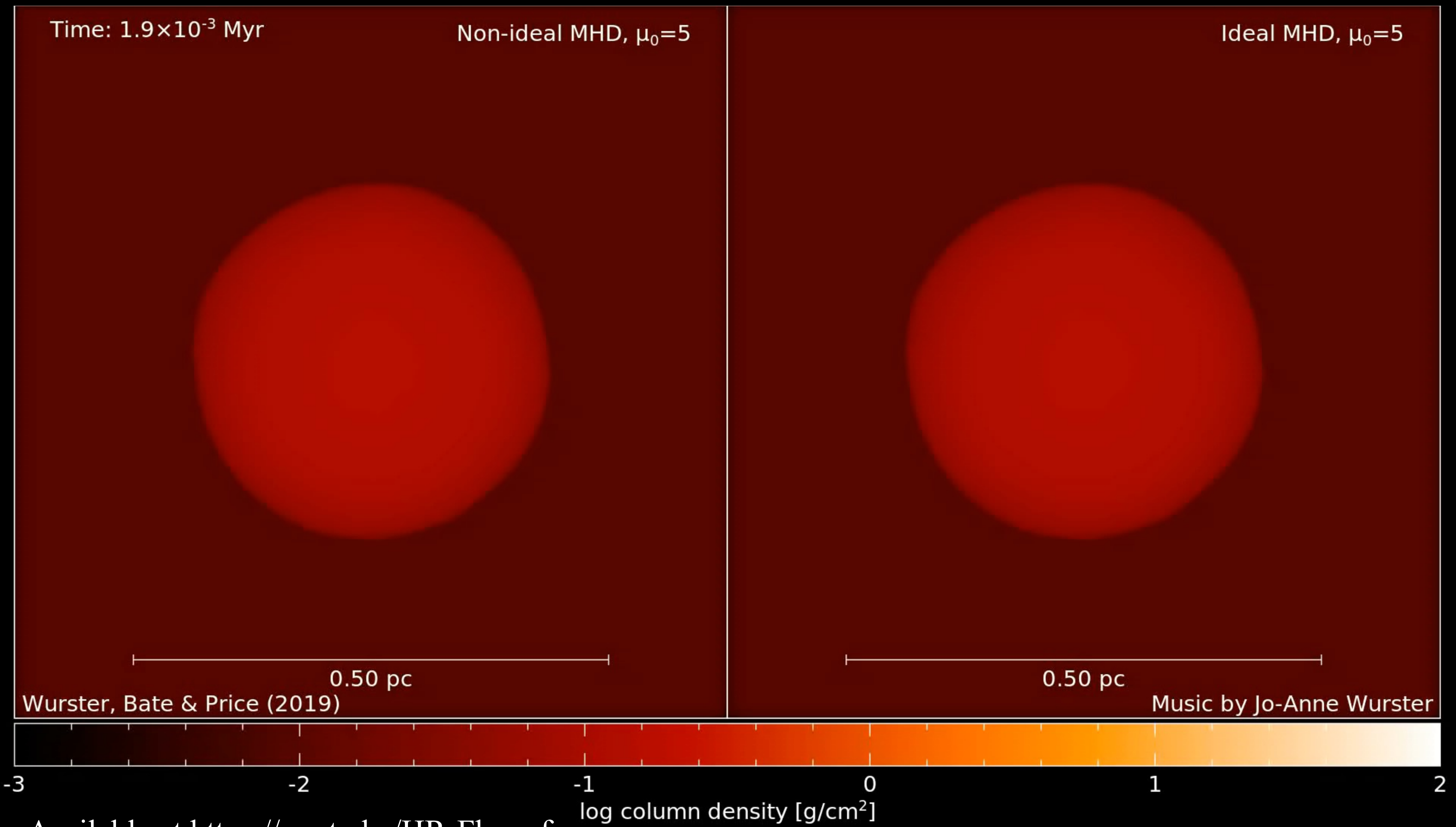
Magnetic fields in star forming regions: Non-ideal Effects



- Cyan lines is typical star forming tracks
- Values dependent on microphysics: Grain size, ionised species, cosmic ray ionisation rate

Adapted from Wardle (2007); constructed using NICIL v2.0 (Wurster, 2016)

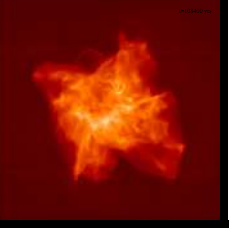
Cluster Formation: Effect of Non-ideal MHD



Available at https://youtu.be/HRrFknoqfac

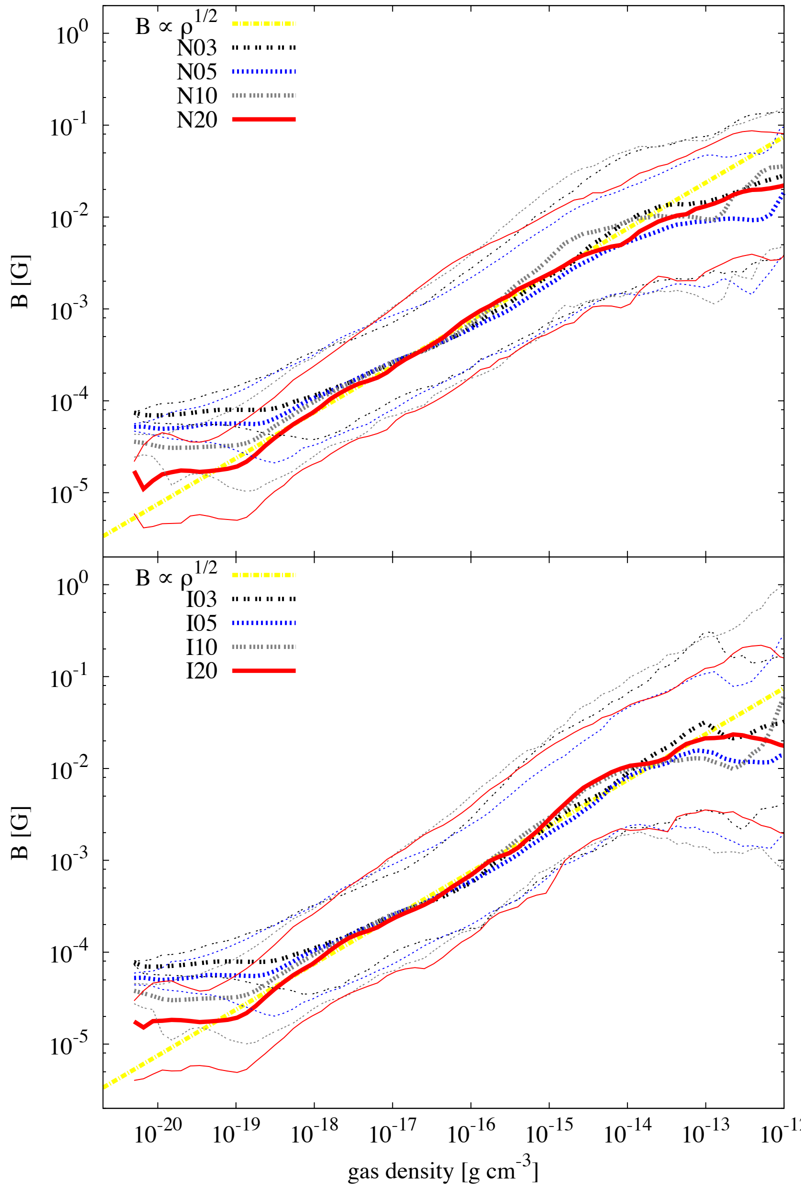
Wurster, Bate & Price (2019)

Music by Jo-Anne Wurster

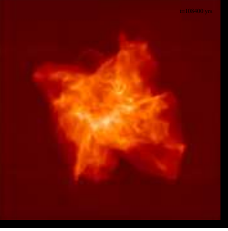


Cluster Formation: Star forming regions

- ➢Star forming regions have a wide range of initial magnetic field strengths, that are approximately independent of the global environment

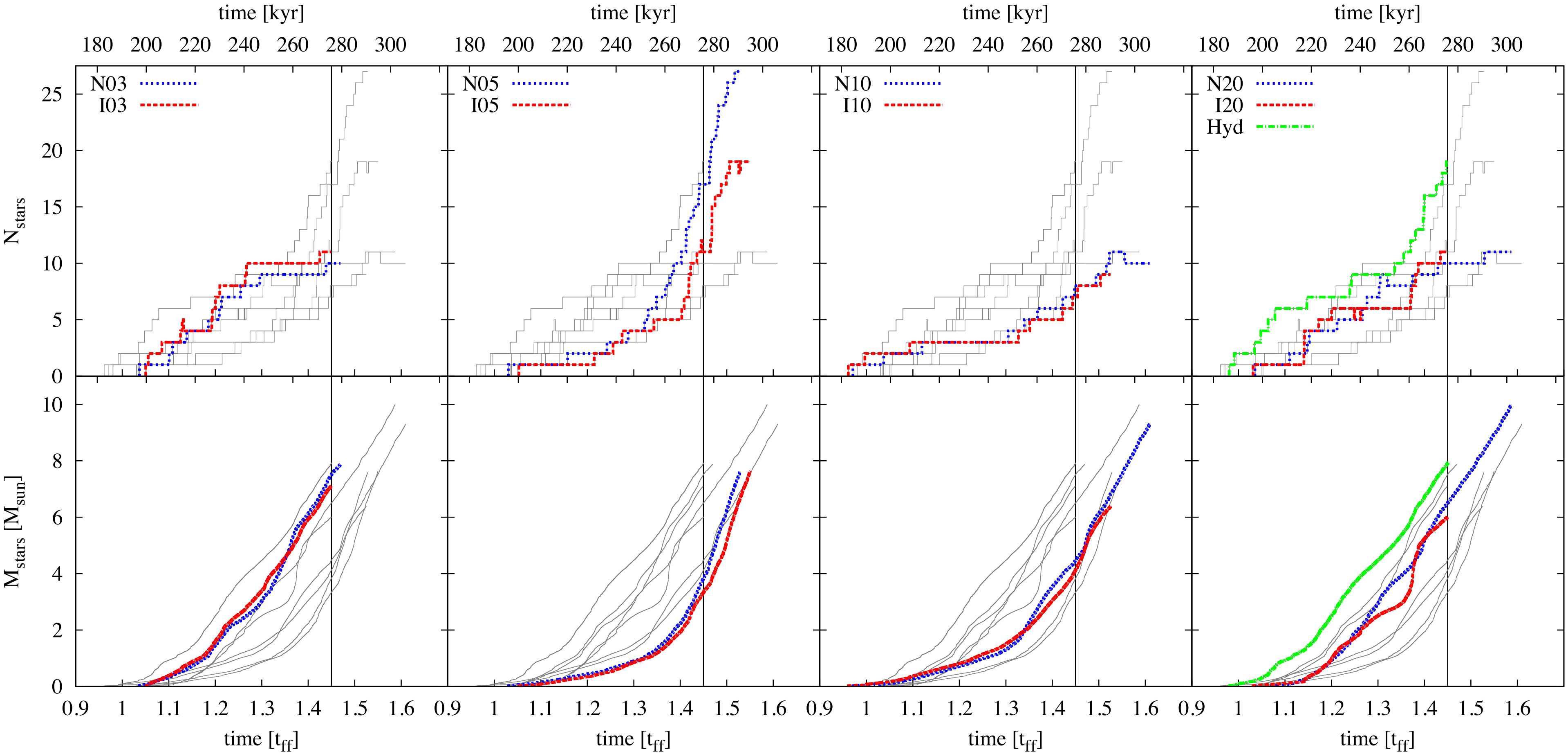


Wurster, Bate & Price (2019)

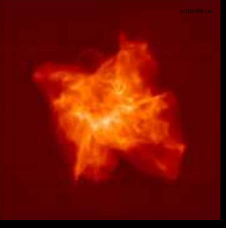


Cluster Formation: Stellar Mass

- ➢No trend when stars form
- ➢Excluding N03 & I03, there is more mass in stars with weaker initial magnetic field strengths

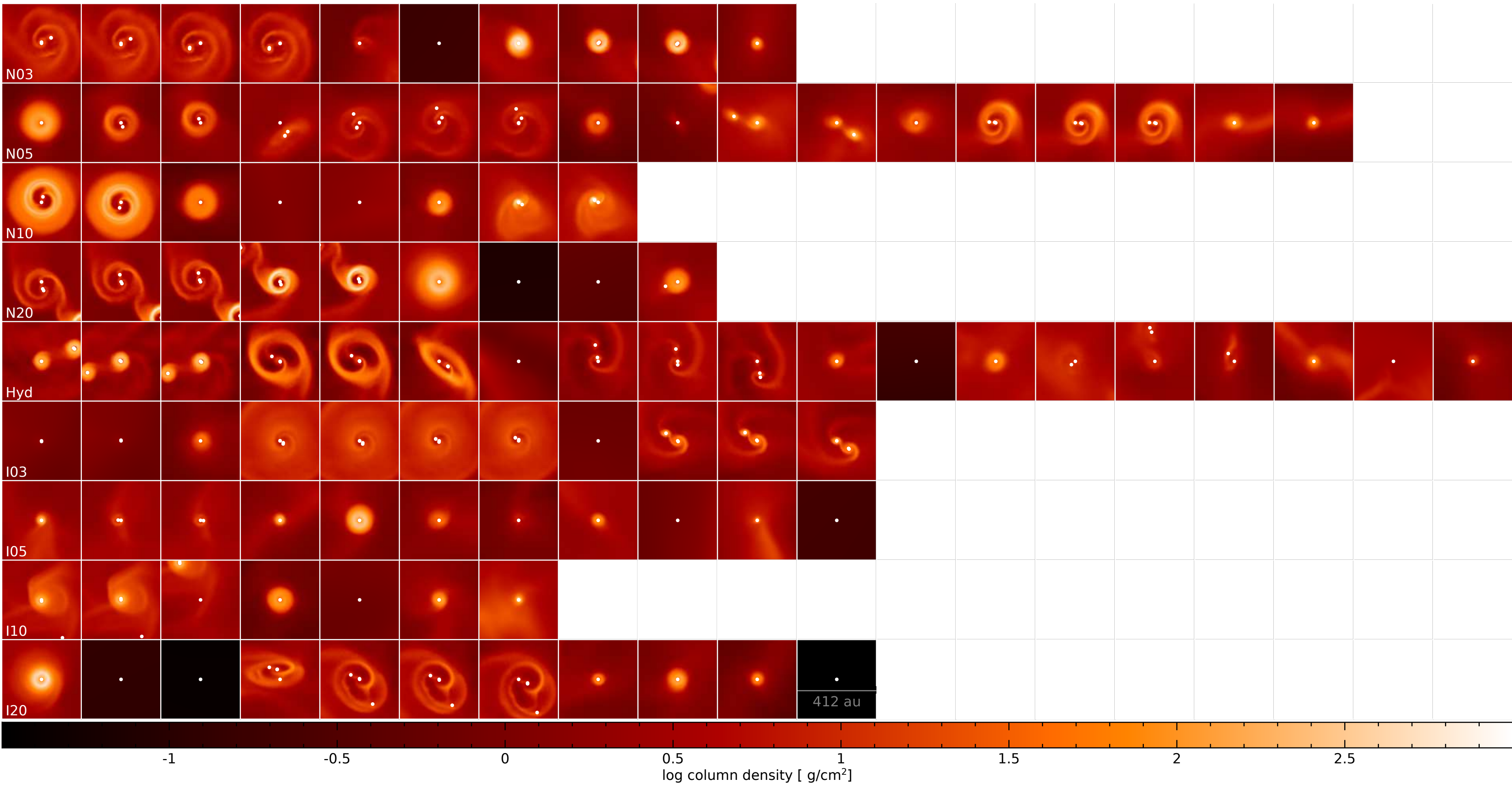


Wurster, Bate & Price (2019)



Cluster Formation: Protostellar discs

- Large protostellar discs form in *all* our models



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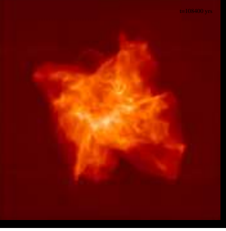

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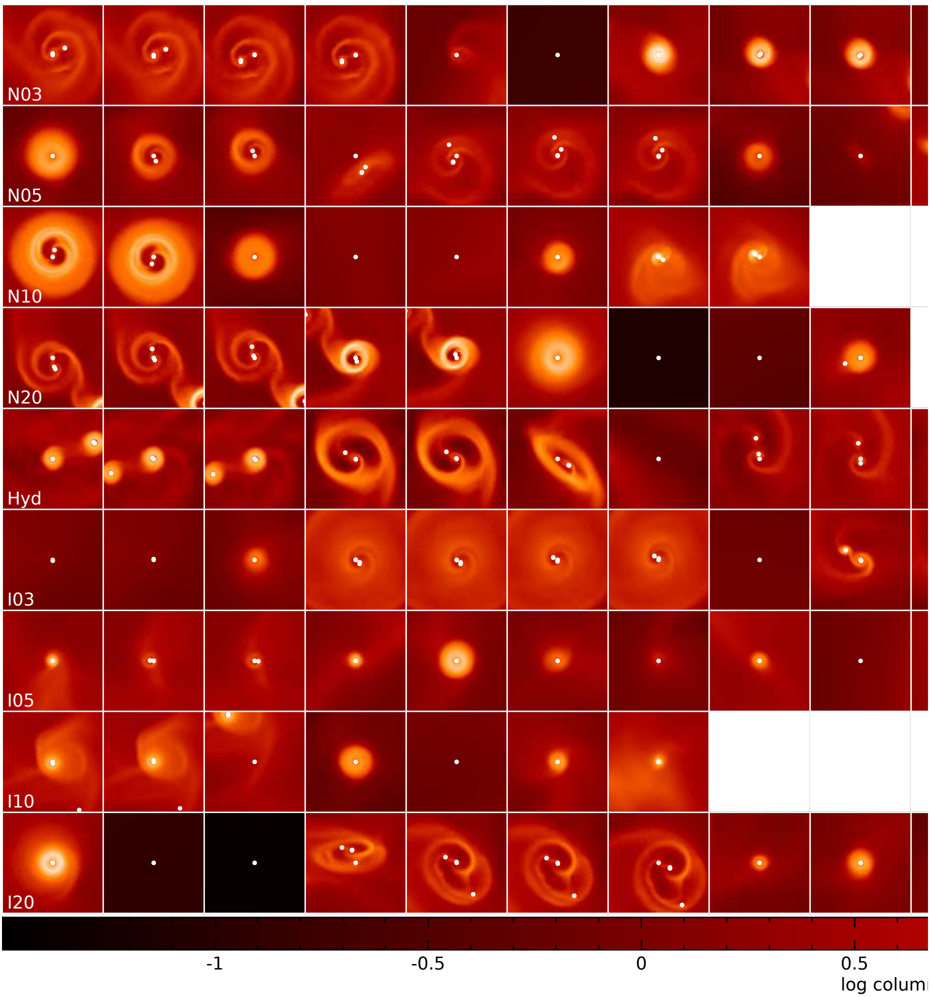
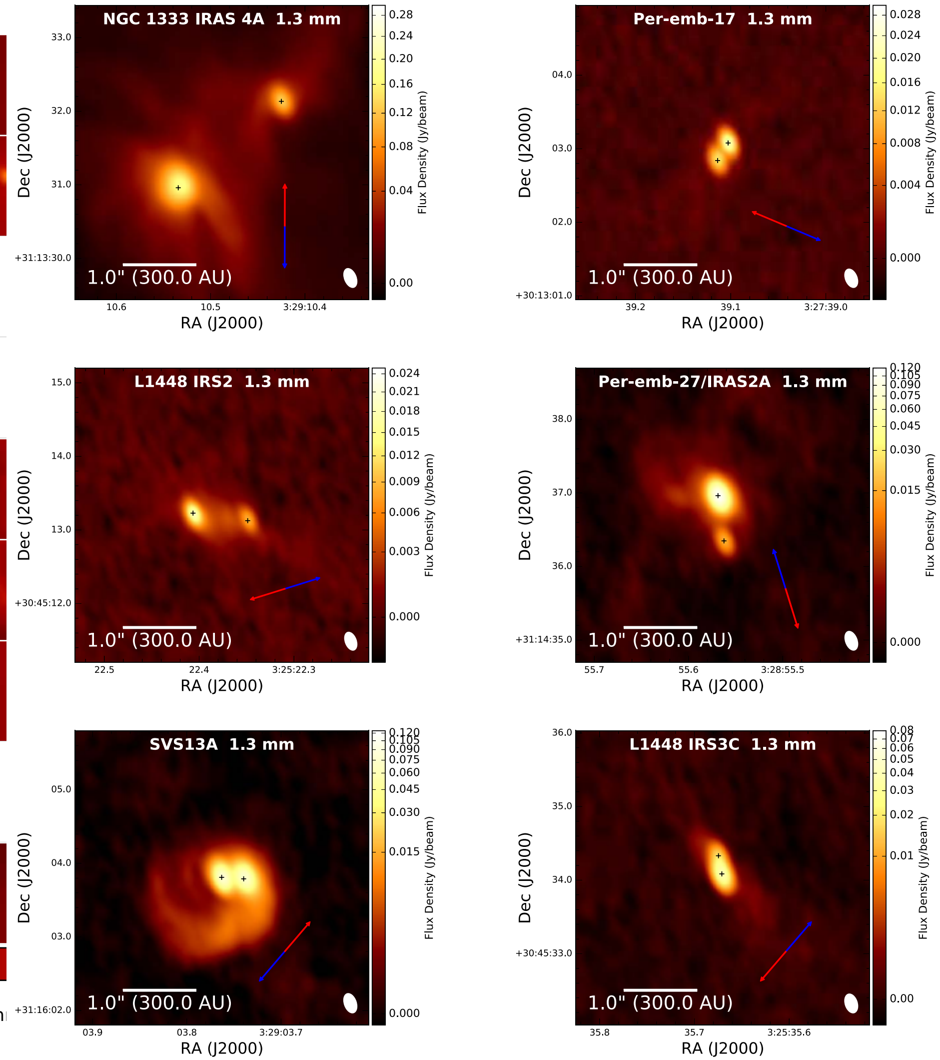


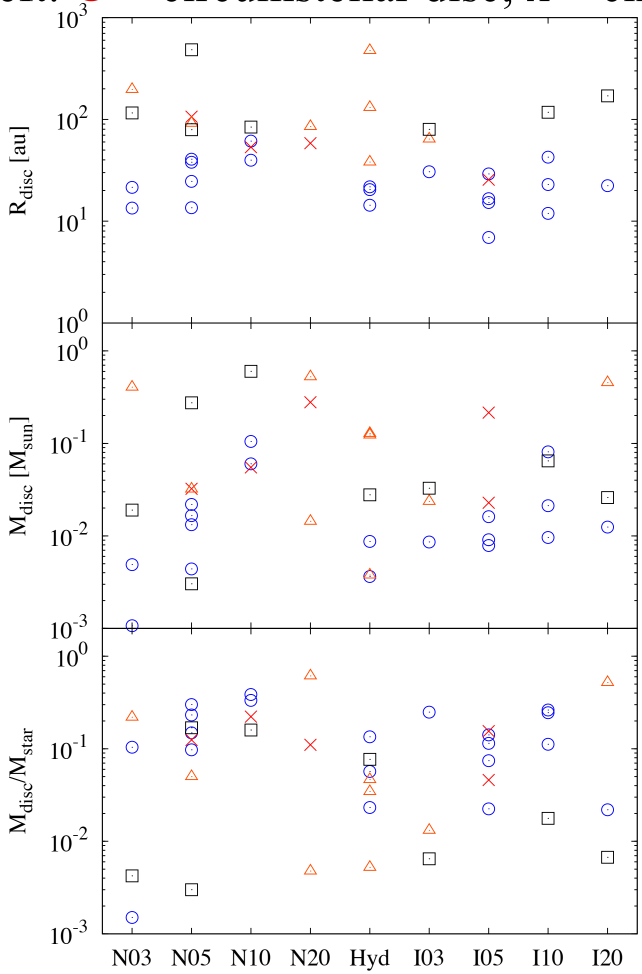


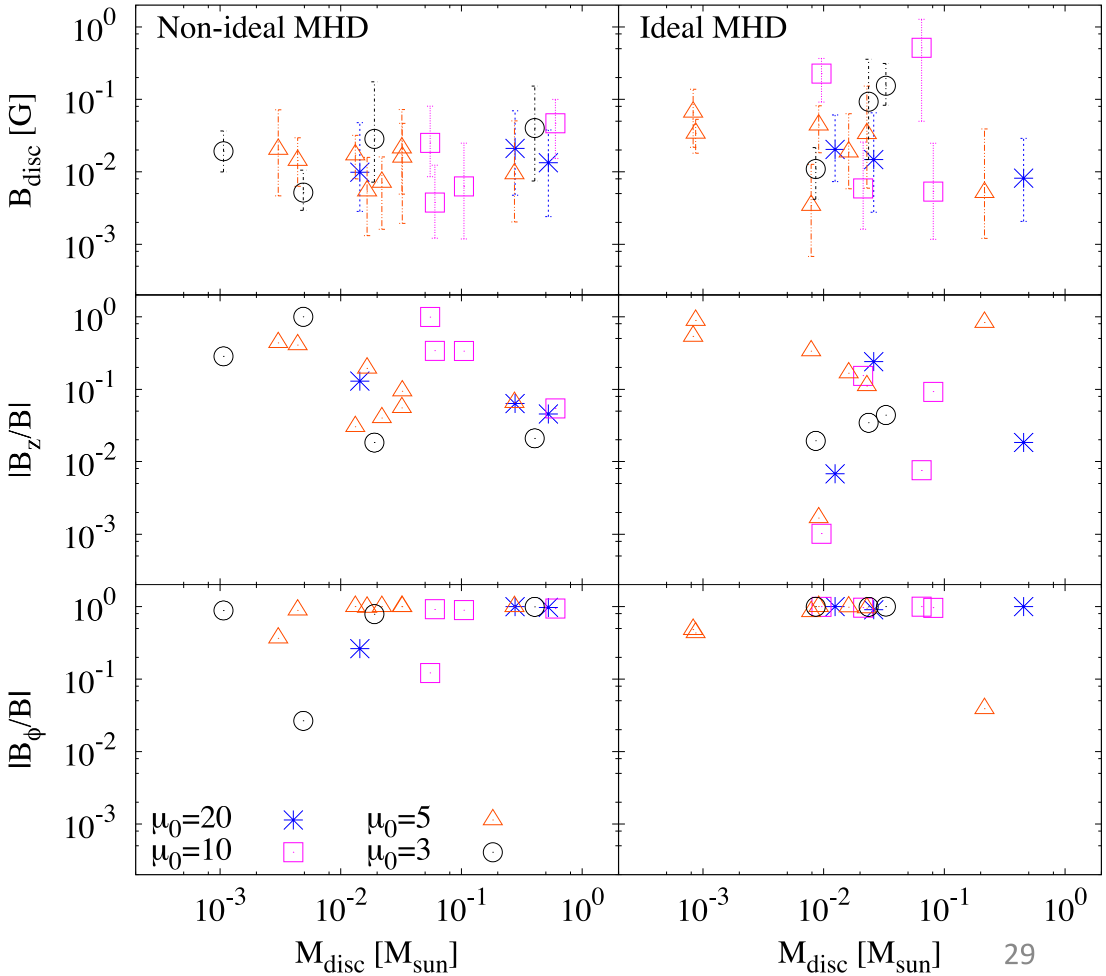
Wurster, Bate & Price (2019)

Discs in Perseus (Tobin+2018)

Cluster Formation: Protostellar discs

- Stellar & disc hierarchy is continuously evolving
- There exist circumstellar discs, circumbinary discs, and circumsystem discs
- Left: ⭕ = circumstellar disc; x = circumbinary disc; △ (🔲)= circumsystem discs about 3 (4) stars





Wurster, Bate & Price (2019)

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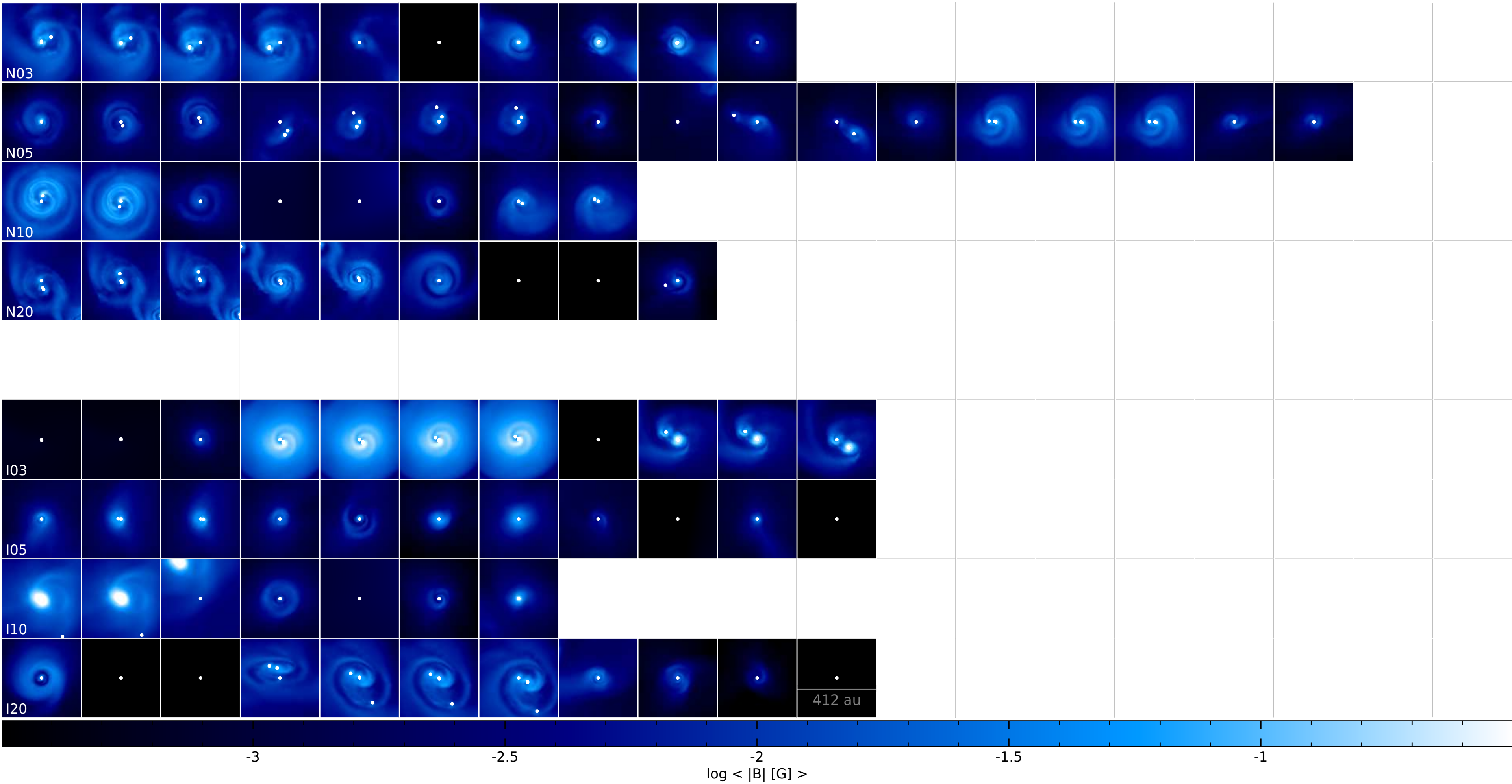
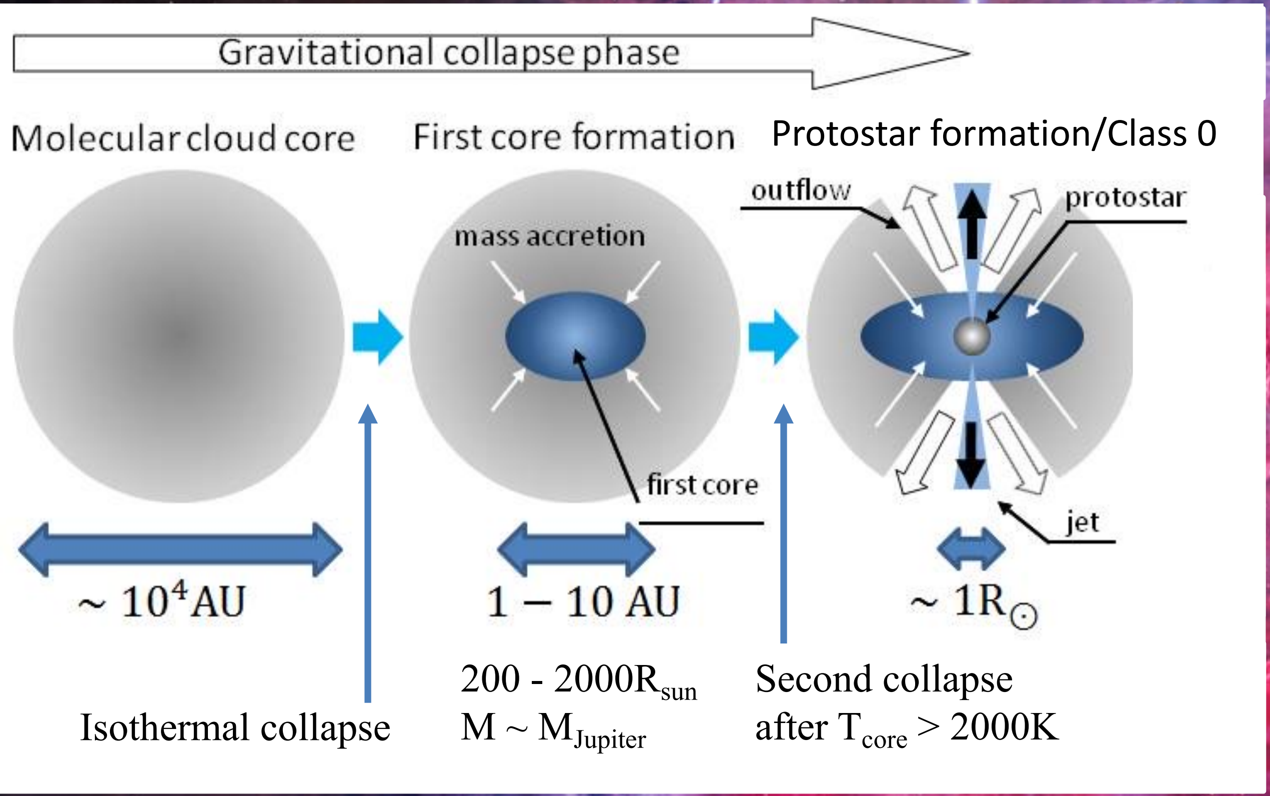


Wurster, Bate & Price (2019)

Star formation: From the beginning

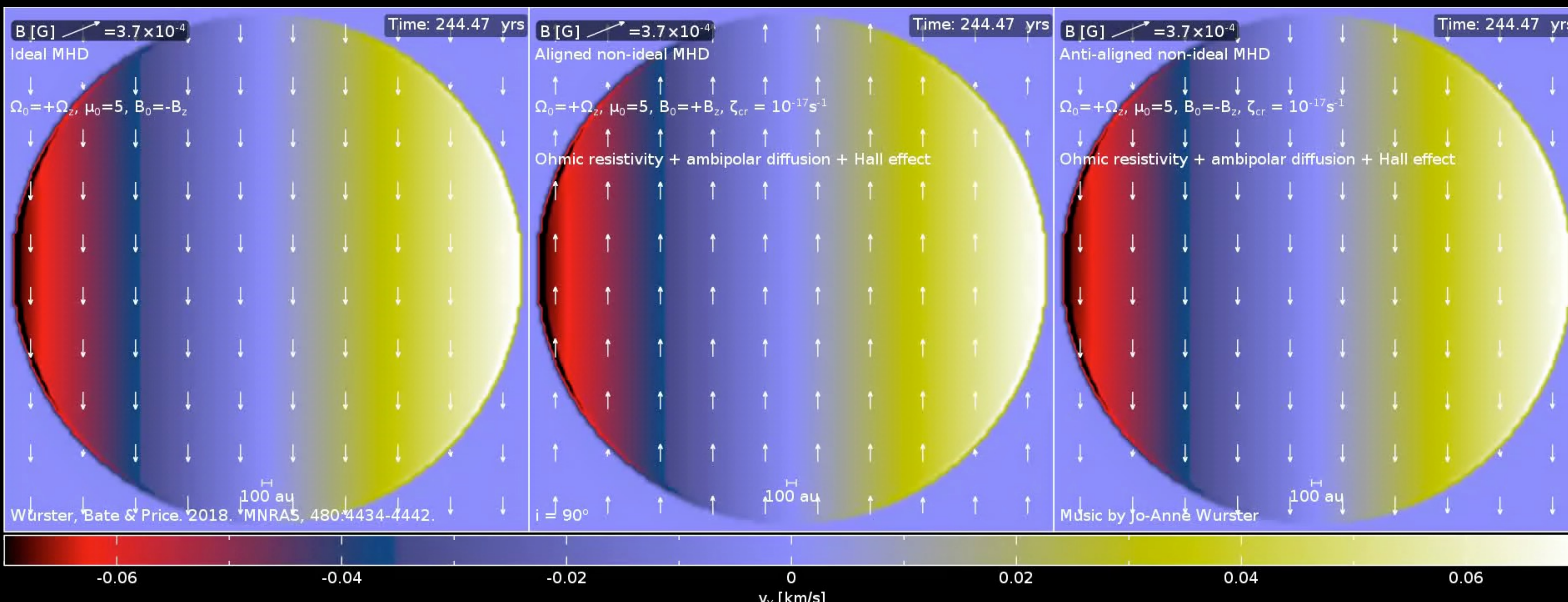


- Relevant processes:
  - Gas
  - Dust
  - Radiation
  - Magnetic fields
  - Kinematics: Rotation
  - Kinematics: Turbulence
  - Etc…

- Disc formation is a natural consequence of star formation

Larson (1969); Illustration by Y. Tsukamoto
Background: Orion Molecular Cloud. (image credit: Drudis & Goldman via APOD)

Formation of a low-mass star



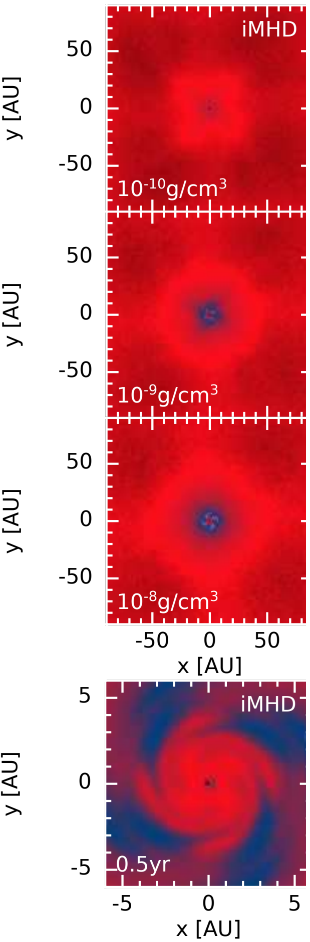
Available at: https://youtu.be/2SQxgXbdJyg

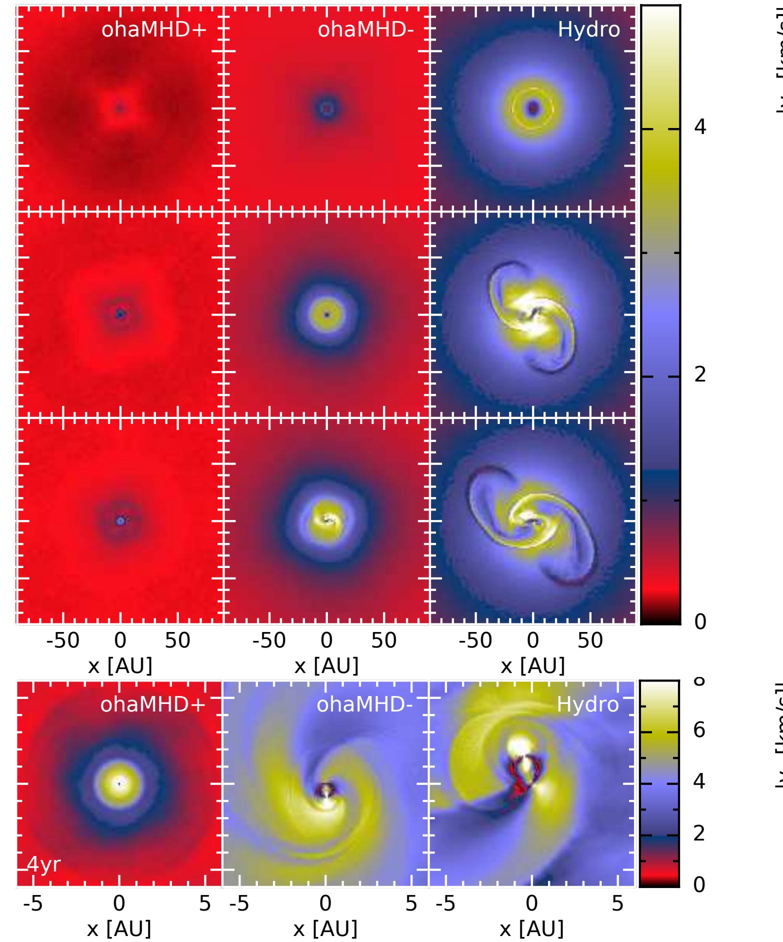
Wurster, Bate & Price (2018c)

Music: Jo-Anne Wurster

Rotationally supported discs

- Discs form in the hydrodynamics model and the non-ideal model with $-B_z$





- Discs form during the first hydrostatic core phase
- Similar disc structure obtained by Tsukamoto+ (2015a) with $\pm B_z$

Wurster, Bate & Bonnell (2021); Wurster, Bate & Price (2018a,c)

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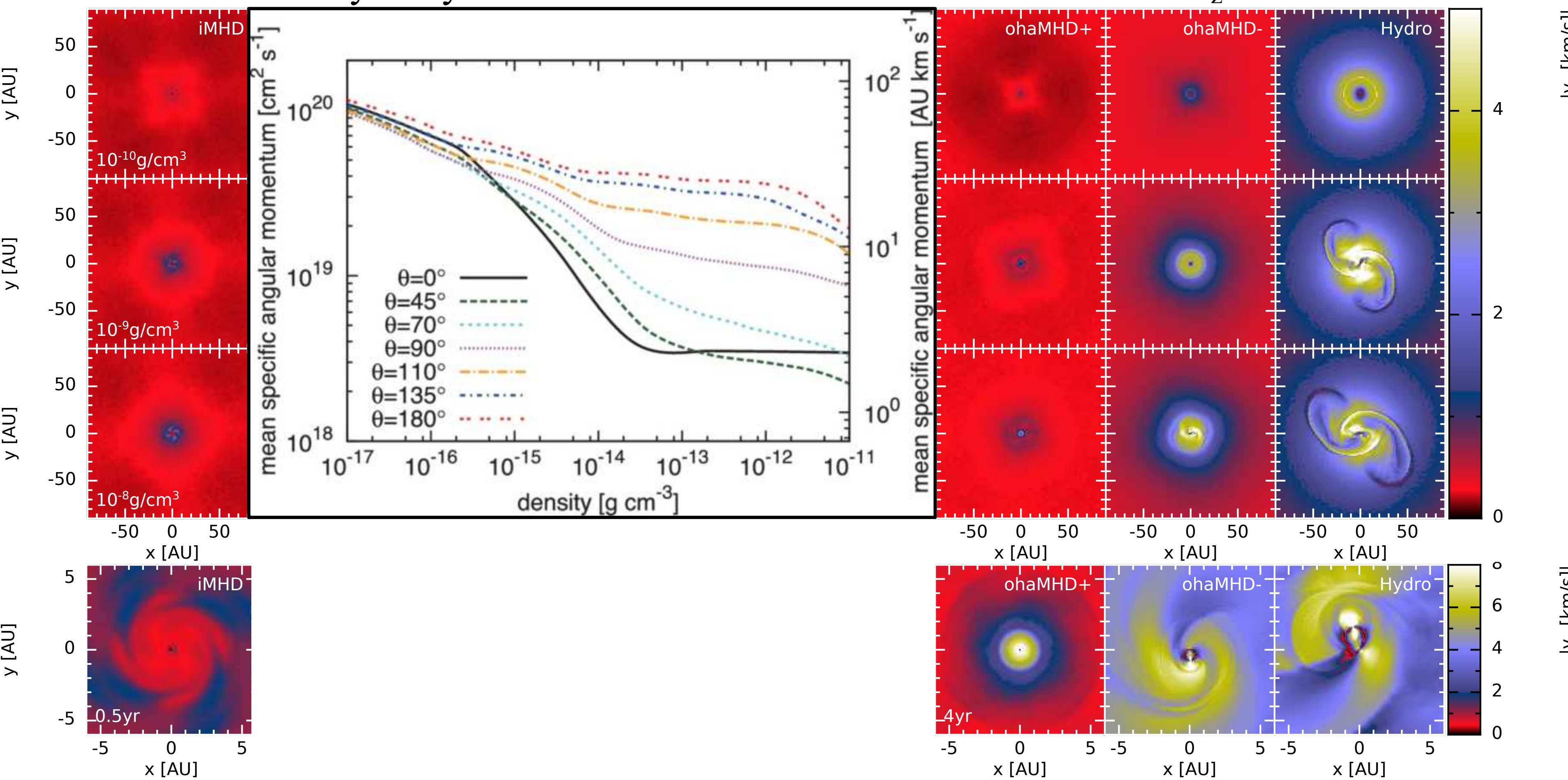

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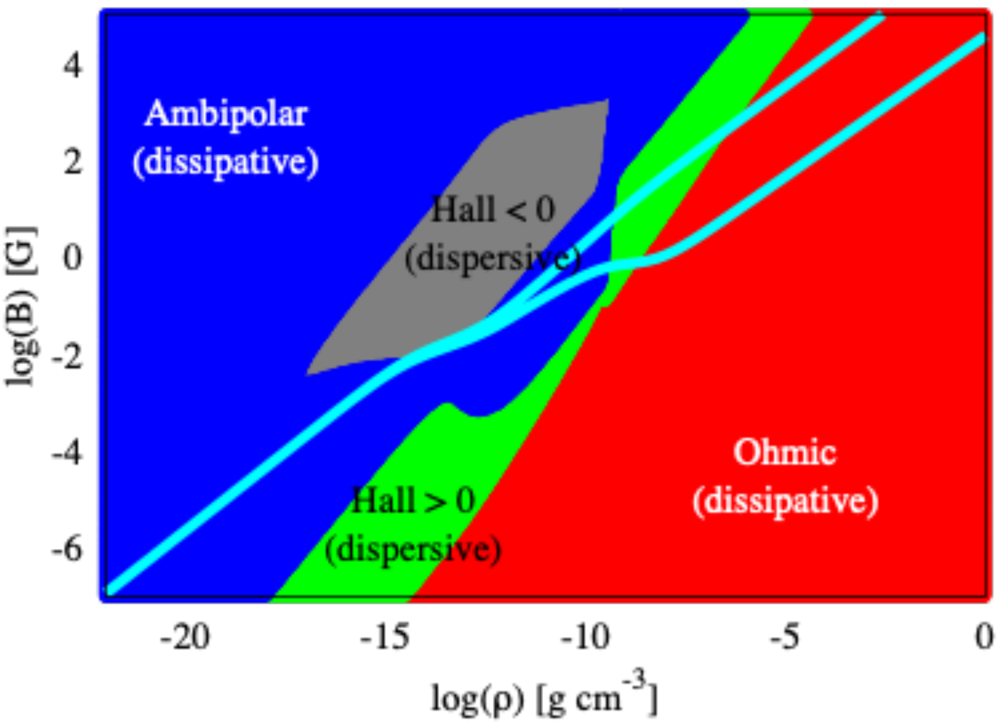
Wurster, Bate & Bonnell (2021); Wurster, Bate & Price (2018a,c); inset: Tsukamoto+ (2017)

Non-ideal magnetohydrodynamics: Components

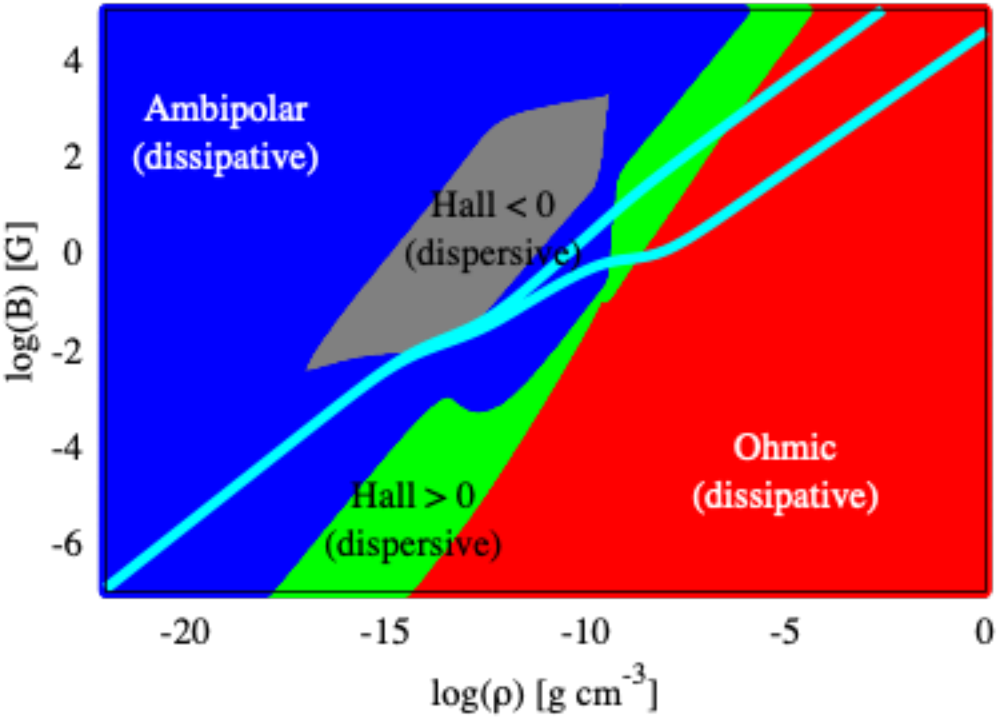


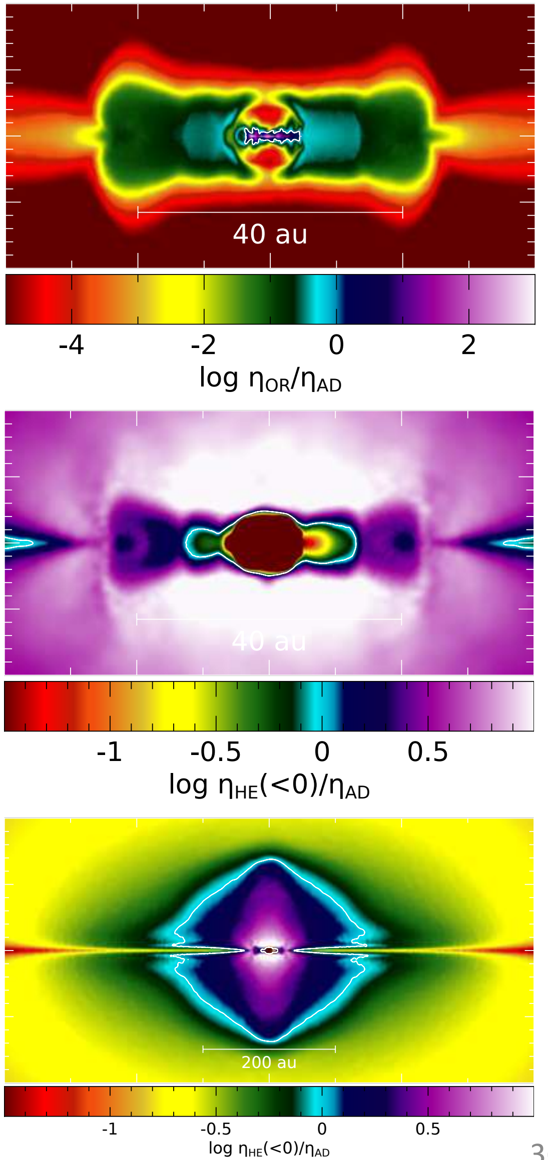
- Multiple conclusions in the literature regarding disc formation with Ohmic resistivity and/or ambipolar diffusion
- Likely possible to form small 1-5au discs in the long term with only Ohmic and/or ambipolar (Dapp and Basu 2010, Machida+ 2011, Dapp+ 2012, Tomida+ 2015, Tsukamoto+ 2015a, Masson+ 2016)
- Hennebelle et al. (2016) predicts 18au discs for ambipolar diffusion only
- Open question: *When do discs form?*

Adapted from Wardle (2007); constructed using NICIL v2.0 (Wurster, 2016)

Non-ideal magnetohydrodynamics: Components

➢Despite the apparent simplified phase space, many processes are important simultaneously, specifically the Hall effect & ambipolar diffusion





Adapted from Wardle (2007); constructed using NICIL v2.0 (Wurster, 2016) Wurster (2021)

Non-ideal MHD Components: Rotationally supported discs

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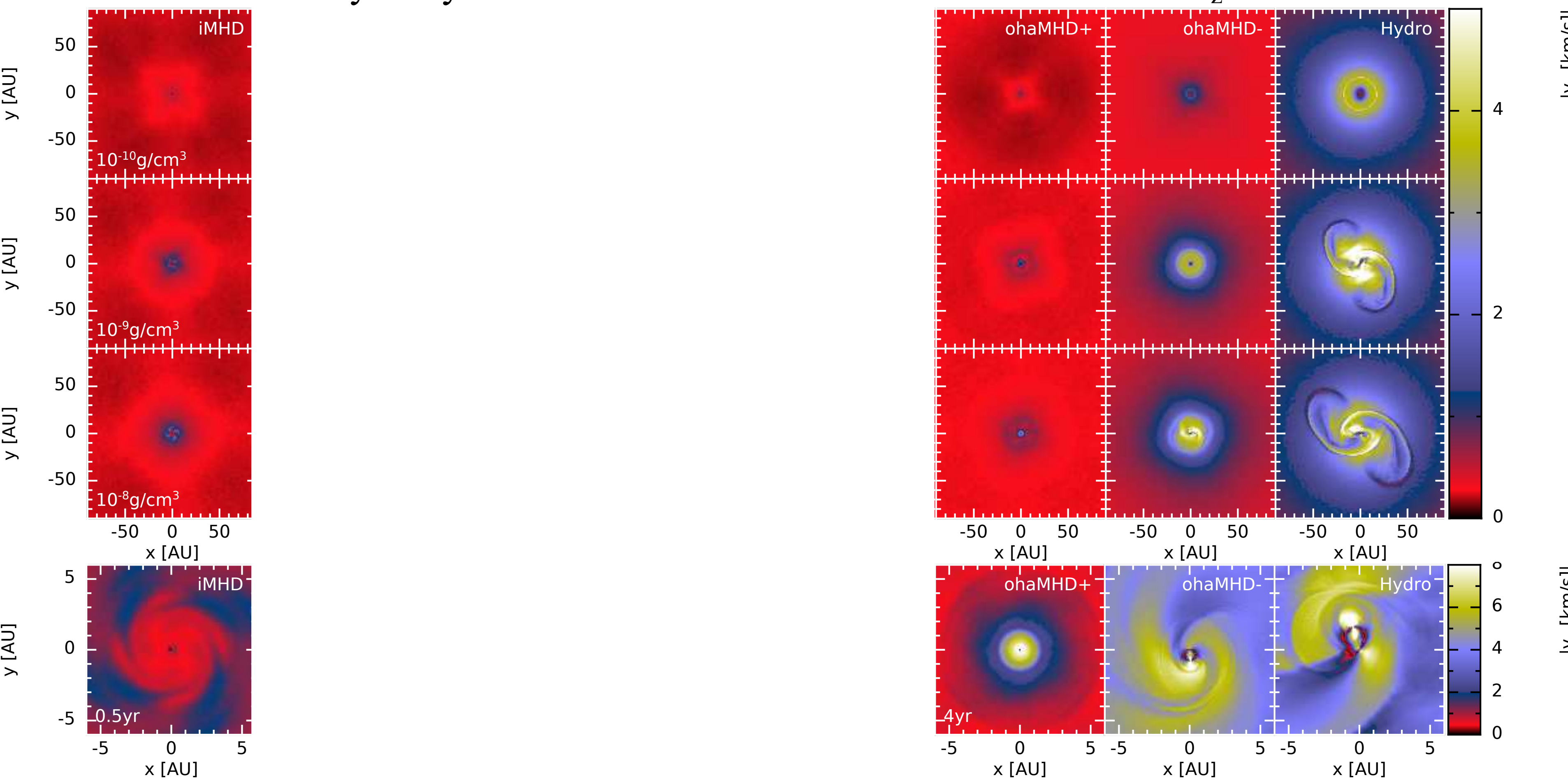

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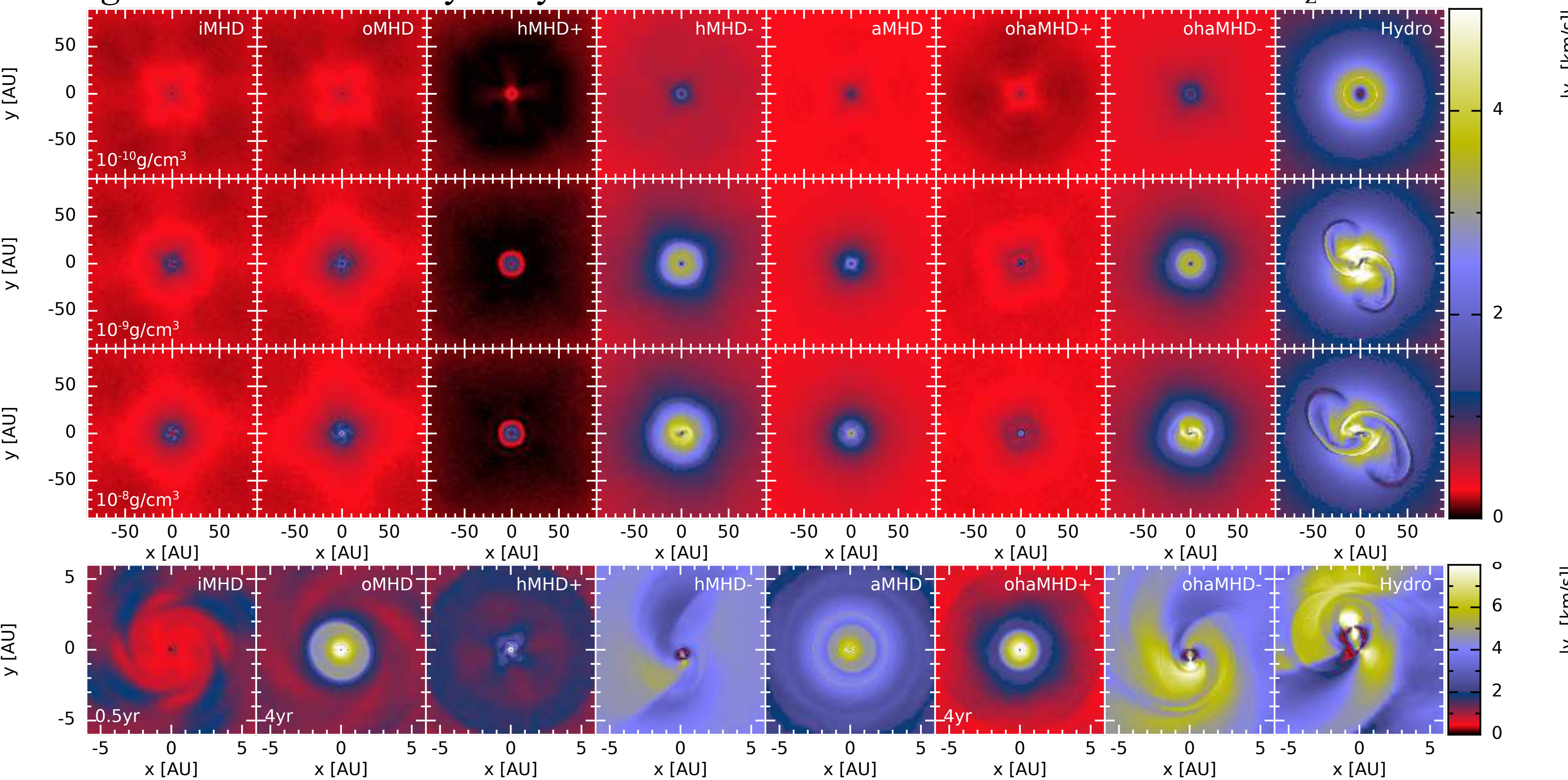
Wurster, Bate & Bonnell (2021); Wurster, Bate & Price (2018a,c)

Non-ideal MHD Components: Rotationally supported discs

- *Large* discs form in the hydrodynamics model and the non-ideal model with $-B_z$

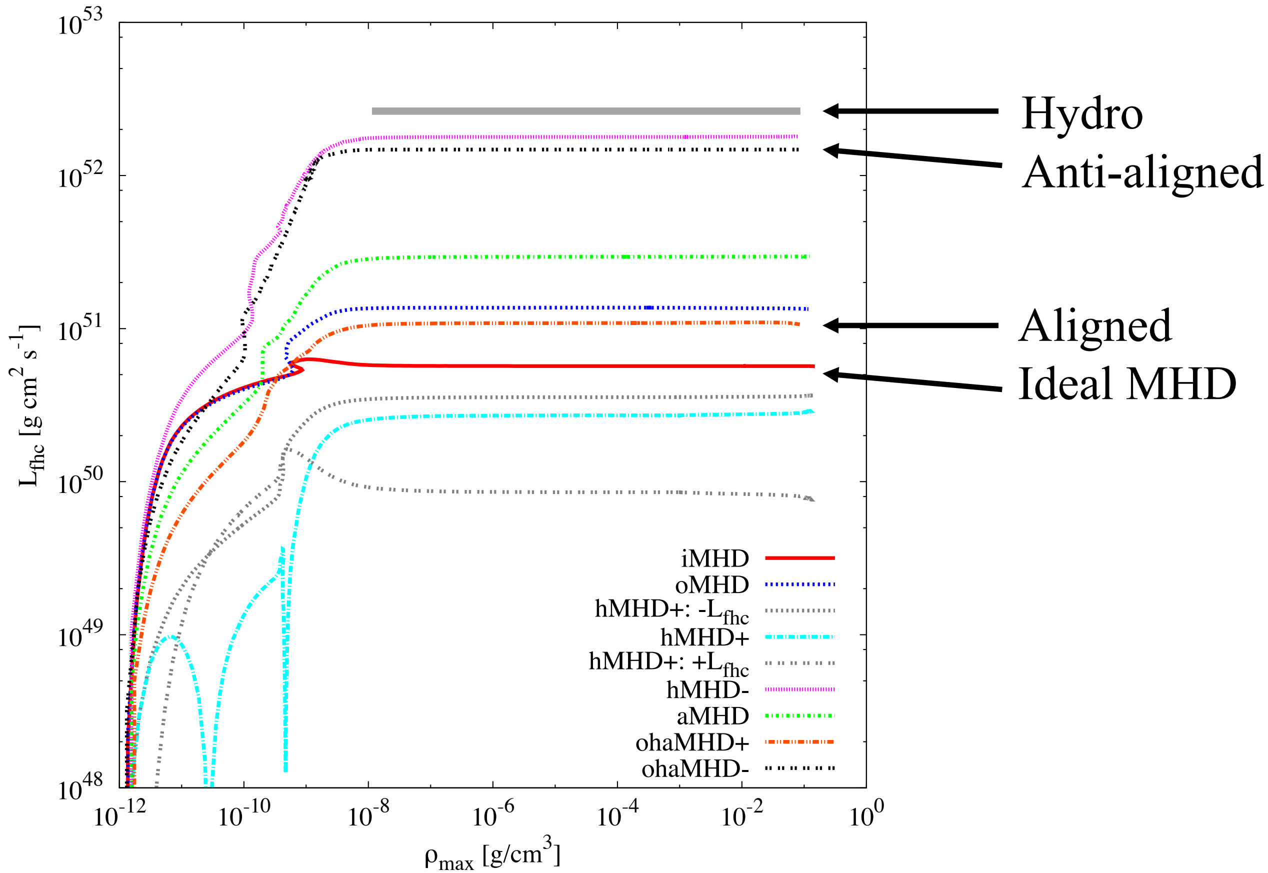


- The Hall effect with $-B_z$ is primarily responsible for forming large discs early
- Ohmic resistivity & ambiploar diffusion will form small discs later

Wurster, Bate & Bonnell (2021); Wurster, Bate & Price (2018a,c); see also Wurster, Price & Bate (2016)

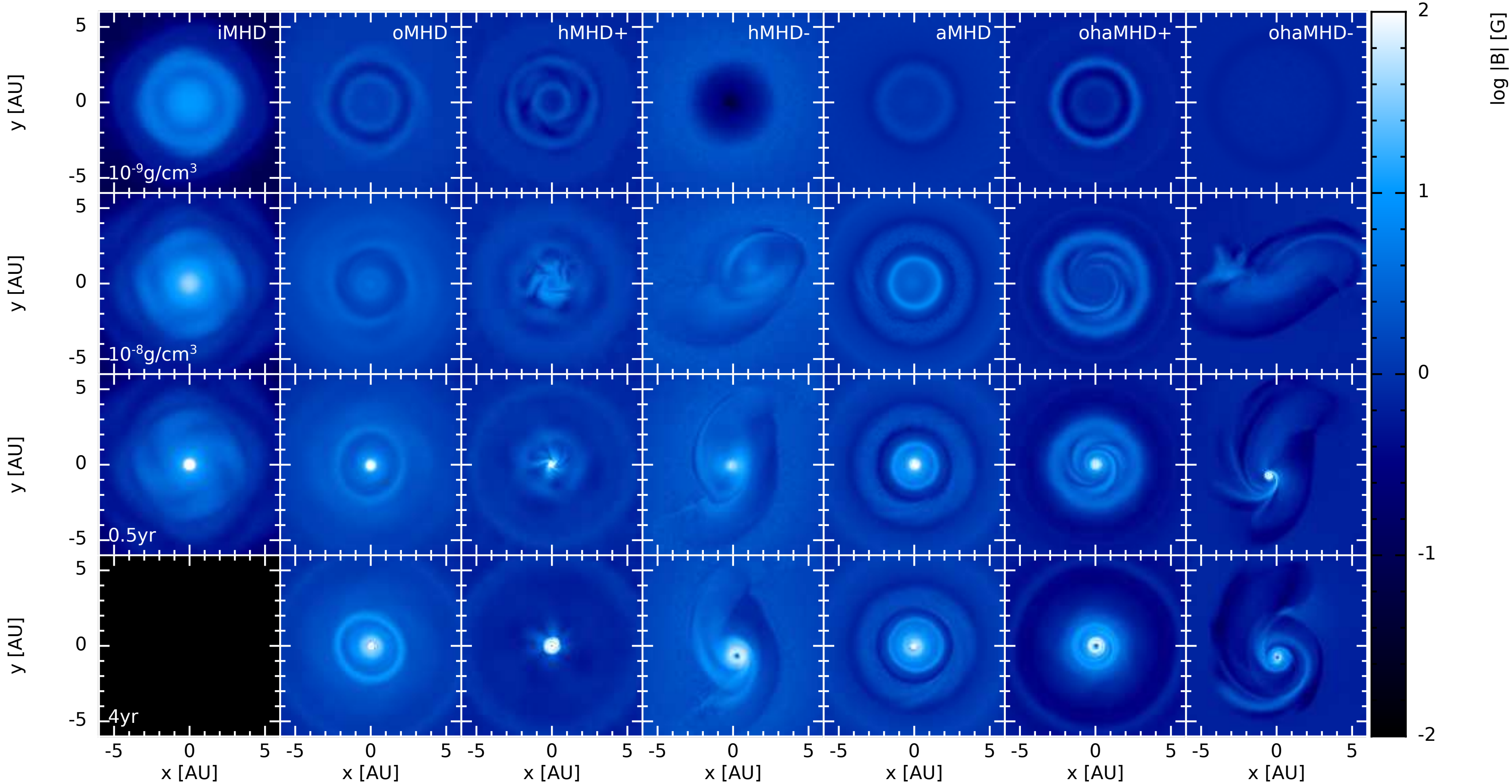
Non-ideal MHD Components: Angular momentum

➢All non-ideal components, except the Hall effect with $+B_z$ increase the angular momentum of the first core, thus promote disc formation



Wurster, Bate & Bonnell (2021)

Non-ideal MHD Components: Magnetic field evolution

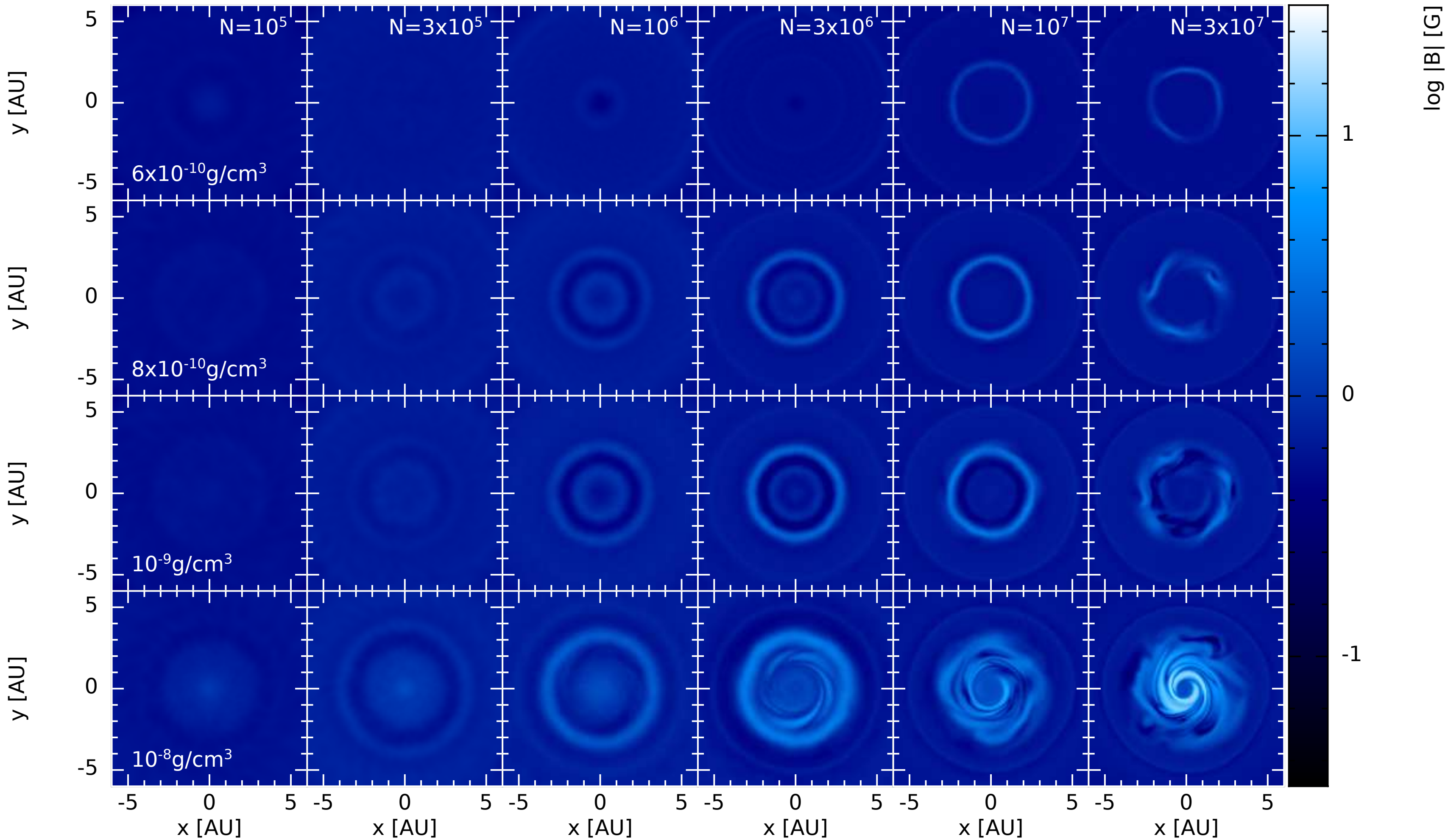


- ➢Magnetic walls (Tassis &Mouschovias, 2005) form in non-ideal MHD models
- ➢Ohmic resistivity & ambipolar diffusion cause the formation of magnetic walls
- ➢The Hall effect creates dispersion that creates spirals

Wurster, Bate & Bonnell (2021); Wurster, Bate & Price (2018d)

Non-ideal MHD Components: Magnetic field evolution

- Magnetic walls (Tassis &Mouschovias, 2005) form in non-ideal MHD models
- Walls are resolution dependent; higher resolutions resolve faster whistler waves



Wurster, Bate, Price & Bonnell (2022)

Conclusions

- Star forming molecular clouds are only weakly ionised
  - Ideal MHD is a poor description

- Star cluster formation:
  - No trends amongst most of our parameters
  - Discs form in all of our models, *suggesting that the magnetic braking catastrophe is a result of poor initial conditions*

- Isolated, low-mass star formation:
  - Large discs only form in the hydrodynamic and non-ideal MHD model with -B_z.
    - *this resolved the magnetic braking catastrophe*
  - All non-ideal MHD terms play a role, with AD & HE the most significant
  - The Hall effect is the primary driver for transporting angular momentum
  - Diffusive / dispersive terms create walls / spirals in the magnetic field

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